(54) Title: ADAPTIVE CONTROL SYSTEM WITH EFFICIENTLY CONSTRAINED ADAPTATION

(57) Abstract

An adaptive control system (110) implements a back–projection technique to limit adaptation of adaptive parameters (yn) in the system so that system actuators (16) are not driven beyond desired physical limitations. When the optimal controller solution lies outside of a desired region in the parameter space, chosen in accordance with the physical limitations of the system, adaptation is back–projected onto or near a smooth convex surface defining the edge of the desired region. Adaptation is preferably normalized to improve adaptation convergence. Back–projection is preferably compensated in accordance with adaptation normalization to facilitate convergence. To lessen computational burdens, adaptation and/or back–projection is accomplished in accordance with a time–sharing technique in which orthogonal components are separately processed. The technique can be implemented in tonal control systems, and in systems capable of controlling non–periodic disturbances.

ADAPTIVE CONTROL SYSTEM WITH
EFFICIENTLY CONSTRAINED ADAPTATION

FIELD OF THE INVENTION

The invention relates generally to adaptive control systems and methods,

5   and more particularly, to active acoustic attenuation systems where constraint of adaptive

parameters defining controller output is desired.

BACKGROUND OF THE INVENTION

The present invention was developed during ongoing research and

developmental efforts by the assignee to improve performance of adaptive control systems.

10   An example of an active acoustic control system developed by the assignee which is

capable of attenuating non-periodic acoustic disturbances is disclosed in U.S. Patent No.

5,621,803 entitled "Active Attenuation System With On-Line Modeling of Feedback Path",

by Trevor A. Laak, issued on April 15, 1997, assigned to the assignee of the present

application, incorporated by reference herein. In many active control applications,

15   cancellation is required only at discrete frequencies where tonal disturbances exist. An

example of an adaptive tonal control systems and methods developed by the assignee is

disclosed in copending U.S. Patent Application Serial No. 08/369,925 entitled "Adaptive

Tonal Control System With Constrained Output and Adaptation", by Steven R. Popovich,

filed on January 6, 1995, now U.S. Patent No. 5,633,795, issued on May 27, 1997 which

20   is incorporated by reference herein.

Problems can sometimes develop in adaptive control systems when the

controller attempts to drive one or more of the actuators (i.e., loudspeakers in a sound

attenuation system) beyond physically sustainable limits. For low or medium actuator

output, the transfer function for actuators is characteristically linear. However, when

25   actuator output becomes high, the actuator transfer function becomes non-linear and the

system can become unstable and/or physical components of the system can be damaged. It

is therefore desirable to constrain controller output so that the maximum output of each

actuator is limited within the linear range of each individual actuator. One way of

constraining controller output involves the use of leakage methods, but leakage methods

30   can compromise overall system performance when used for the purpose of limiting output

power. Examples of power limiting using leakage techniques include the system disclosed

in copending patent application Serial No. 08/553,186 entitled "Frequency Selective Active

Adaptive Control System", by Shawn K. Steenhagen et al., filed on November 7, 1995,

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assigned to the assignee of the present application, now U.S. Patent No. _____; and U.S. Patent No. 5,627,896 entitled "Active Control of Noise and Vibration", by Steve C. Southward et al., issued on May 6, 1997.

5   In many active control applications, it is necessary to use multiple inputs and multiple outputs to attain effective control. The use of high numbers of sensors and actuators along with sophisticated adaptation schemes can stretch computational requirements beyond those practical. It is therefore not only important that adaptation converge reliably to an adequate solution, but also that adaptation occurs efficiently within realistic signal processing requirements.

10   The filtered-X algorithm is an effective means for controlling disturbances at multiple locations when there are a relatively small number of sensors and actuators. However, as the number of actuators and error signals becomes large, convergence rates tend to slow. Normalizing adaptation to provide more direct convergence improves tracking in tonal systems, and also benefits performance in feedforward systems cancelling

15   random disturbances.

It is desirable to provide normalized adaptation for quick convergence while at the same time limiting individual actuator output so that the effectiveness of each individual actuator is maximized, all without exceeding reasonable signal processing resources provided by conventional digital signal processors used for active acoustic

20   attenuation. It is also important for the limiting of the actuator outputs to be performed in a manner which is compatible with the normalization of the adaptation so that these functions may be performed simultaneously.

SUMMARY OF THE INVENTION

The invention is an adaptive control system and method that effectively

25   constrains adaptation so that system actuators are not driven beyond one or more selected physical limits. Adaptation is constrained by defining a constraint surface in the parameter space of the adaptive parameters, and directly constraining adaptation when unconstrained adaptation would cause one or more of the adaptive parameters to lie substantially outside of the desired region of adaptation contained within the constraint surface.

30   The invention is preferably implemented using a parameter back-projection technique to constrain adaptation of the adaptive parameters (e.g. FIR filter tap weights in a broadband system, or scaling vectors in a tonal system) when unconstrained

adaptation would cause one or more of the adaptive parameters to lie substantially outside of the constraint surface. The back-projection technique is especially effective because it allows adaptation to migrate along the constraint surface until an optimum solution within or substantially near the constraint surface. It is normally preferred that adaptation be normalized to improve the rate of convergence. When using normalized adaptation, back-projection should be compensated to account for adaptation normalization and to ensure that continued back-projected adaptation seeks the optimum solution for constrained adaptation.

In order to simplify the back-projection procedure and ensure proper convergence of the constrained adaptation, it is desired that the constraint surface be defined a smooth convex surface. If adaptation step size and transformations to compensate for normalized adaptation are chosen properly, the constraint surface can be approximated by a plane that is tangent to the smooth convex surface. Back-projection can then be accomplished to the tangent plane approximating the constraint surface rather than the constraint surface itself. Over time, the position and orientation of the plane changes as constrained adaptation causes the adaptive parameter values to migrate along the constraint surface. In addition, it may be desirable to globally scale the adaptive parameters or otherwise account for differences between the tangent plane and the constraint surface caused by curvature of the constraint surface.

In most applications, it is preferred that the constraint surface be a pre-selected, fixed surface in the parameter space for the adaptive parameters. However, if reference signal statistics for the acoustic disturbance being attenuated or controlled are non-stationary, it may be desirable to define the constraint surface in the adaptive parameter space as a function of reference signal statistics.

Inasmuch as normalized adaptation can require significant signal processing capabilities due to matrix operations, it may be desirable to perform adaptation in accordance with a time-sharing technique. Thus, in another aspect, the invention involves the use of a convenient time-sharing technique in which unconstrained update signal vectors are accumulated over a plurality of sample periods. Linearly independent components of the accumulated update vector are extracted individually from the accumulated update vector, and the extracted linearly independent component is used for constrained adaptation of the adaptive parameters. Preferably, the linearly independent

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components are orthogonal components which are determined through a decomposition of the covariance matrix for a filtered version of the reference signal or the C path matrix. Normalization of adaptation as well as back-projection is accomplished independently for each component by back-projecting and scaling the respective component used for

5   constrained adaptation on the adaptive parameters. In this manner, computational burdens are significantly reduced, which is especially important in high-dimensional systems. System performance is not compromised as long as each individual linearly independent component is extracted and processed within a reasonable time frame.

The invention can be embodied in a system designed to attenuate or

10   control tonal disturbances such as the system disclosed in U.S. Patent Application Serial No. 08/369,925 entitled "Adaptive Control System With Constrained Output and Adaptation", by Steven R. Popovich, now U.S. Patent No. 5,633,795, issued on May 27, 1997, which utilizes normalized adaptation and null space constraint to optimize system performance. The invention can also be used in a system capable of attenuating or

15   controlling non-periodic disturbances, for instance a system which preferably operates as disclosed in U.S. Patent No. 5,621,803, entitled "Active Attenuation System With On-Line Modeling of Feedback Path" by Trevor A. Laak, which uses a recursive adaptive filter model. Details of these systems are described in conjunction with the following drawings.

Other features and aspects of the invention may be apparent to those

20   skilled in the art upon inspecting the following drawings and description thereof.

<div align="center">BRIEF DESCRIPTION OF THE DRAWINGS</div>

Adaptive Tonal Control System

Fig. 1a is a schematic illustration of an active acoustic attenuation system that attenuates a tone at a discrete frequency in accordance with copending U.S. Patent

25   Application Serial No. 08/369,925, now U.S. Patent No. 5,633,795.

Fig. 1b is a detailed schematic illustration of the system shown in Fig. 1a.

Fig. 2 is a graphical illustration of the difference between a convergence path for gradient descent adaptation and a convergence path for normalized adaptation.

Fig. 3a is a schematic illustration of an active tonal attenuation system

30   with back-projected adaptation in accordance with the invention.

Fig. 3b is a detailed schematic illustration of the system shown in Fig. 3a.

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Fig. 4 is a graphical illustration of the convergence of a normalized parameter update combined with uncompensated back-projection.

Fig. 5 is a graphical illustration of back-projected adaptation in which the back-projection is compensated for normalized adaptation.

5   Fig. 6 is a vector diagram of back-projected adaptation for limiting actuator output in accordance with the invention.

Fig. 7 is a graphical illustration of using a smooth convex constraint surface that represents the combined constraint surface for two actuators in the system.

Fig. 8a is a plot depicting the rate at which a system in accordance with 10   the invention converges.

Fig. 8b is a graph illustrating the magnitude of outputs from each of a plurality of actuators in a system operating in accordance with the invention.

Fig. 9a is a schematic illustration of another embodiment of an active tonal attenuation system with back-projected adaptation to limit actuator output in 15   accordance with the invention.

Fig. 9b is a schematic illustration of another embodiment of an active tonal attenuation system implementing a time-sharing technique.

Adaptive Broadband Control System

Fig. 10 is a schematic illustration of an active acoustic attenuation system 20   that is capable of attenuating or controlling a non-periodic acoustic disturbance in accordance with U.S. Patent No. 5,621,803.

Fig. 11 is a schematic illustration of the system shown in Fig. 10 implementing back-projected adaptation in accordance with the invention.

Fig. 12 is a graphical illustration of a typical two-dimensional constraint 25   surface and system error performance contours mapped in the parameter space of the adaptive parameters.

Fig. 13 is a vector diagram of back-projected adaptation for limiting actuator output in accordance with the invention.

Fig. 14 is a schematic illustration showing a constraint surface in the 30   parameter space combining the efforts of two separate constraint functions.

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DETAILED DESCRIPTION OF THE DRAWINGS

Adaptive Tonal Control System

Fig. 1a illustrates an active acoustic attenuation system 10 in accordance with above incorporated, U.S. Patent Application Serial No. 08/369,925 entitled "Adaptive Tonal Control System With Constrained Output And Adaptation", by Steven R. Popovich, now U.S. Patent No. 5,633,795, issued on May 27, 1997. The system 10 uses an adaptive controller 12 to attenuate a tone at a particular frequency in a disturbance 18. The adaptive controller 12 is preferably embodied within a programmable digital signal processor. The adaptive controller 12 has an adaptive parameter bank 13, a parameter update generator 28; and an error weighting element 26. To attenuate several tones at distinct frequencies, several attenuation systems 10 such as shown in Figs. 1a and 1b can be implemented separately and contemporaneously on the same digital signal processor. Separate tones are substantially orthogonal so an adaptive controller 12 implementing separate and contemporaneous tonal attenuation systems 10 can effectively attenuate several tones in a disturbance 18.

In the adaptive controller 12, the adaptive parameter bank 13 generates a plurality of n correction signals y_n. Each of the n correction signals y_n drives an actuator 16 that provides a secondary input or cancellation signal 17 that combines with a system input to yield a system output 21. That is, the secondary inputs 17 from the actuators 16 propagate into the system and attenuate the disturbance 18 to yield the system output 21 as represented schematically by summing junction 20. A plurality of p error sensors 22 senses the system output 21, and generates p error signals e_p. In Fig. 1a, the path of the n correction signals y_n through the n actuators 16, the path of the secondary inputs or cancellation signals between the actuators 16 and the error sensors 22, and the path through the p error sensors 22 is defined as a p x n C path (e.g. a p x n speaker-error path), and is illustrated by block 24.

The adaptive controller 12 receives an error signal e_p from each of the p error sensors 22. The controller 12 has an error weighting element 26 (i.e. an n x p matrix) that processes the p error signals e_p to yield n error input signals e.

The parameter update generator 28 in the controller 12 receives the n error input signals e, and generates a set of parameter updates u. The parameter updates u are used to adapt one or more scaling vectors in the adaptive parameter bank 13. The

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scaling vectors are adapted by accumulating the updates u with the existing scaling vector. The scaling vector is then typically applied to a tonal reference signal to generate the n correction signals y_n.

Also, in accordance with the copending patent application, now U.S. Patent No. 5,633,795, the error weighting element 26 is chosen to improve the convergence of the adaptation process. There are several methods for generating the error weighting element 26, but it is preferred that a C model of the C path 24 be used to generate the error weighting element 26. The C model can be generated off-line, but it is preferred that the C model be adaptively generated on-line as described in U.S. Patent No. 4,677,676 which is incorporated herein by reference for the purposes of adaptive on-line C modeling. In the system 10, the C model is a p x n matrix, where the ij^{th} element represents the complex frequency response of the pathway from the jth output channel to the output of the ith error sensor at the frequency of the disturbance.

The error sensors 22 preferably generate error signals e_p every sample period k. It is desirable to adapt the controller 12 rapidly in real time with respect to sample period k. This can be approximated over time by demodulating the error input signals e by the in-phase and quadrature components of the particular frequency being attenuated. The demodulation is accomplished using in-phase and quadrature demodulation signals in the parameter update generator 28. The in-phase and quadrature components are formed for the particular frequency being attenuated.

Fig. 1b illustrates in detail the system 10 shown in Fig. 1a. In Fig. 1b, the controller 12 receives an input signal x(k) from an input sensor 30. The input signal x(k) is transmitted to a phase-locked loop circuit 32 in the controller 12. The phase-locked loop circuit 32 outputs a reference signal at a particular frequency which is the frequency of the tone being attenuated. In particular, the reference signal is preferably a discrete time sequence in the form of a cosine wave at a particular frequency. It is preferred that the reference signal have a normalized (e.g. unity) magnitude.

The reference signal is separated into two signals at junction 34: An in-phase reference signal is transmitted through line 36, and a quadrature reference signal is transmitted through line 38. The in-phase reference signal is transmitted through line 36 to an in-phase scaling element 40. The in-phase scaling element 40 multiples the in-phase reference signal by an in-phase scaling vector Y_R (i.e. an adaptive parameter vector) to

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generate n in-phase components y_r of the adaptive output signals y_n. The in-phase scaling element 40 stores the values of the in-phase scaling vector Y_R, and updates the values. In U.S. Patent No. 5,633,795, the values of Y_R are updated by summing the product of an in-phase update signal u_r multiplied by a convergence step size μ.

Contemporaneously, quadrature components y_i of the output signals y_n are generated. The quadrature reference signal is transmitted through line 38 to a phase shifter 42 that shifts the quadrature reference signal 90° to in effect generate a sine wave corresponding to the cosine wave. Thus, in this context, the term quadrature reference signal corresponds to a reference signal that has been phase shifted 90° from the in-phase reference signal. The quadrature scaling element 44 multiplies the quadrature reference signal by a quadrature scaling vector Y_I (i.e. an adaptive parameter vector) to generate m quadrature components y_i of the adaptive output signals y_n. The scaling element 44 stores the values of the quadrature scaling vector Y_I, and updates the values. In U.S. Patent No. 5,633,795, the values of Y_I are updated by summing the values by the product of a quadrature update signal u_i multiplied by the step size μ.

The n in-phase output signals y_r and the n quadrature y_i output signals are summed at summer 46 to generate n correction signals y_n. The n correction signals y_n are transmitted to n actuators 16.

The error weighting element 26 is determined using the p x n C matrix to eliminate problems associated with over-parameterization and to also account for phase shifts and delay in the auxiliary C path 24. In accordance with U.S. Patent No. 5,633,795, the C matrix can be decomposed at the frequencies of interest using singular value decomposition as represented below:

$$C = USV^H \quad (1A)$$

where U is a p x p matrix, S is a p x n matrix, and V^H is an n x n hermitian transpose of an n x n matrix V. The matrices U and V are unitary matrices, and the off diagonal elements of S are zero while the diagonal elements are in general real and positive. The error weighting element 26 applies an n x p matrix $H_2 = BC^H$, where $B = VN^HNV^H$, V is the n x n matrix defined in equation (1A); N^H is an n x p matrix that is the hermitian transpose of normalizing matrix N which is formed by inverting some of the values on the

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diagonal of S (e.g., the values that are not zero or close to zero). Setting $B = I$ (identity matrix) results in a gradient descent update. The use of transformation matrix B is to compensate the gradient descent update, thus creating a normalized update which improves the rate of convergence by providing a more direct adaptation path.

5   Error weighting element 26 preferably has a junction 48, an in-phase weighting element 50 and a quadrature weighting element 52. Each of the p error signals e_p is transmitted to the junction 48, and the p error signals e_p are then contemporaneously transmitted to the in-phase weighting element 50 and to the quadrature weighting element 52. The in-phase element 50 of the error weighting element 26 contains the real parts of

10   the complex elements of the error weighting matrix H_2. The quadrature element 50 of the error weighting element 26 contains the coefficients of the imaginary parts of the complex elements of the error weighting matrix H_2. Both the in-phase 50 and the quadrature 52 elements of the error weighting element 26 contain real values. When referring herein to in-phase and quadrature weighting element, the term in-phase weighting element refers to

15   the real parts of the complex elements in a weighting matrix, and the term quadrature weighting element refers to the imaginary parts of the complex elements in a weighting matrix. The p error signals e_p are processed contemporaneously through the in-phase element 50 and the quadrature element 52 to each provide n error input signals e. Both sets of n error input signals are real, and are transmitted to the update generator 28.

20   The update generator 28 includes junctions 54 and 60, multipliers 56, 58, 62 and 64, and summers 66 and 68. The set of n error input signals e from the in-phase element 50 of the error weighting element 26 is transmitted to junction 54, where the signals e are split. From junction 54, one set of n error input signals e is provided to multiplier 56, and another set of n error input signals e is provided to multiplier 58.

25   Likewise, the set of n error input signals e from the quadrature element 52 of the error weighting element 26 is transmitted to junction 60, where the signals e are split. From junction 60, one set of n error input signals e is provided to multiplier 62, and another set of n error input signals e is provided to multiplier 64.

          The n error input signals e provided to multiplier 62 are multiplied by the

30   in-phase demodulation signal 70, which is preferably the same as the normalized in-phase reference signal 36. The n error input signals e provided to multiplier 56 are multiplied by the quadrature demodulation signal 72, which is preferably the same as the normalized

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phase-shifted quadrature reference signal in line 43. This demodulation should occur during each sample period of adaptation. The output from multipliers 56 and 62 is summed in summer 66 to generate the negative of n updates u_i for the quadrature scaling vector Y_1 in the quadrature scaling element 44 that generates the quadrature components y_i
5   of the output signals.

The n error input signals e provided to multiplier 58 are multiplied by the normalized in-phase demodulation signal 76. The n error input signals e provided to multiplier 64 are multiplied by the normalized quadrature demodulation signal 74. This demodulation should occur during each sample period of adaptation. The output from
10   multipliers 58 and 64 is subtractively summed in summer 68 to generate n updates u_r for the in-phase scaling vector Y_R in the in-phase scaling element 40 that generates the n in-phase reference signals y_r.

As mentioned earlier, the scaling vectors Y_R and Y_1 are the adaptive parameters in the adaptive parameter bank 13. In U.S. Patent No. 5,633,795,
15   unconstrained update signals u_r and u_i are used to adapt the scaling vectors Y_R and Y_1, respectively. Each scaling vector Y_R and Y_1 contains n components.

Referring now to Fig. 2, the use of transformation matrix B improves the rate of convergence, and improves the performance of the system 10. Fig. 2 illustrates representative adaptation trajectories in a system having two actuators 16 for a normalized
20   update 76 in contrast to a gradient descent update 78. For simplicity of illustration, the plot in Fig. 2 shows the real part of two scaling vectors Y_R, and assumes that quadrature scaling vector $Y_1 = 0$. The plot in Fig. 2 shows quadratic error performance surface contours (i.e., contours representing level of error cost function) for an optimal solution depicted by star 80. The box shown in bold represents a constraint surface S for the
25   system 10. This constraint surface encloses the intersection for the interiors of two distinct constraint functions S_1 and S_2 relating to a first and second actuator, respectively. In particular, S_1 represents a limit for the absolute value of the adaptive parameter $Y_{R,1}$ and S_2 represents a limit for the absolute value of the adaptive parameter $Y_{R,2}$. The actuators 16 have a generally linear response inside of the constraint function S. If the adaptive
30   parameter values exist outside of S, at least one of the constraint functions S_1 or S_2 will be violated. In this case the actuator response may become nonlinear and damage or instability may result. Fig. 2 illustrates a situation in which the optimal solution 80 lies

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within the constraint surface S for both actuators 16. Note that under these conditions, the normalized update 76 converges to the same optimal solution 80 as the gradient descent update 78, but the trajectory of the normalized update 76 follows a more direct path towards the optimal solution 80 in contrast to the less direct path of the gradient descent

5 update 78. The adaptation trajectory of the gradient descent update 78 is orthogonal to the performance surface contours. The trajectory of the gradient descent update 78 is different than the trajectory of the normalized update 76 unless the eigenvalues for the matrix product $C^H C$ are equal. Therefore, when the optimal solution 80 lies within the constraint surface S, the normalized update 76 provides the same solution 80 as the gradient descent

10 update 78, but normally does so at a faster rate of convergence, thereby improving system 10 performance.

Occasionally, the optimal solution 80 lies outside of the constraint surface S, which means that if allowed to adapt in the absence of any constraint, the adaptive control system 12 would attempt to drive at least one of the actuators 16 beyond its

15 physical capabilities. Under such conditions, the secondary input or cancellation signal 17 from the actuator 16 might not be commensurate with the correction signal y_n received by the actuator 16 from the adaptive parameter bank 13. This is potentially damaging or unstable. Fig. 3a shows an adaptive control system 110 having a parameter back-projection element 82 for constraining adaptation to prevent these conditions in accordance

20 with the invention.

Referring to Fig. 3a, the purpose of the parameter back-projection element 82 is to constrain adaptation of adaptive parameters (e.g., scaling vectors Y_R, Y_I) in the adaptive parameter bank 13 so that no correction signal y_n exceeds its selected limit. Like reference numbers are used to describe the adaptive tonal control system 110 shown in

25 Fig. 3b as were used in describing system 10 in Fig. 1a where appropriate to facilitate understanding.

The system 110 in Fig. 3a has an adaptive controller 112 to attenuate a tone at a particular frequency in a disturbance 18. The adaptive controller 112 includes an adaptive parameter bank 113, a parameter back-projection element 82, an error weighting

30 element 126, and a parameter update generator 128. To attenuate several tones at distinct frequencies, several attenuation systems 110 can be implemented separately and

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contemporaneously on the same digital signal processor, or on two or more networked digital signal processors.

In the adaptive controller 112, the adaptive parameter bank 113 generates a plurality of n correction signals y_n. Each of the n correction signals y_n drives an actuator 16 that provides a secondary input or cancellation signal 17 that combines with a system input to yield a system output 21. That is, the secondary input 17 from the actuator 16 propagate into the system and attenuate the disturbance 18 to yield the system output 21 as represented schematically by summing junction 20. A plurality of p error sensors 22 senses the system output 21 and generates p error signals e_p. The combined path of the n correction signals y_n through the n actuators 16, from the actuators 16 to the error sensors 22, and through the p error sensors 22, is defined as a p x n auxiliary C path (e.g. a p x n speaker-error path), and is illustrated schematically by block 24.

The adaptive controller 112 receives an error signal e_p from each of the p error sensors 22. The error weighting element 126 processes the p error signals e_p to yield n error input signals e. The error weighting element 126 is preferably an n x p matrix. In this embodiment, the error weighting element 126 applies an n x p matrix $H_2 = BC^H$, where C^H is the hermitian transpose of the p x n C matrix representing speaker-error path 24, and B is an n x n transformation matrix defined by $B = V\overline{\Lambda}V^H$ where matrix V is determined in accordance with an eigenvalue decomposition of $C^H C$, V is an n x n unitary matrix, V^H is the hermitian transpose of matrix V, Λ is a real diagonal matrix containing the eigenvalues of $C^H C$, and $\overline{\Lambda}$ is formed by inverting non-trivial diagonal entries of Λ down to an inversion limit defined in relation to the maximum eigenvalue.

If the dimensions of the system 110 are not large, the above processing matrices (e.g. matrices C, $\overline{\Lambda}$, B, V etc.) are likely to be realizable in a single processor having realistic processing capacity because it is necessary to have C path information only at the one or more discrete frequencies of interest for cancellation.

The parameter update generator 128 in the controller 112 receives the n error input signals e, and generates a set of unconstrained updates u. The unconstrained updates u are used to adapt the adaptive parameters (i.e., scaling vectors Y_R and Y_I) in the adaptive parameter bank 113 as discussed with respect to Figs. 1a and 1b without modification, unless such adaptation requires that one of the correction signals y_n drive a respective actuator 16 substantially beyond the constraint surface S. In accordance with

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the invention, the parameter back-projection element 82 generates back-projection signals
that are combined with the unconstrained update signals u to constrain adaptation of the
adaptive parameters with respect to the constraint surface S defined in the parameter space
of the adaptive parameters (e.g. scaling vector Y_R, Y_I). In other words, the constraint
5   surface S surrounds a desired region for adaptation in the parameter space of the adaptive
parameters. Adaptation of the adaptive parameters is constrained so that none of the
adaptive parameters lie substantially outside of the desired region in the parameter space.
In Fig. 3a, the parameter back-projection element 82 is shown to operate collectively on
the adaptive parameter bank 13 and the parameter update generator 28 contained within
10   dashed block 29. This is meant to illustrate that parameter back-projection can be
accomplished either on the updated adaptive parameters (i.e. Y_R, Y_I) or on the parameter
updates u.

Fig. 3b illustrates in detail a system 110a which is a version of the system
110 shown in Fig. 3a. In the system 110a shown in Fig. 3b, the parameter back-projection
15   element 82 operates specifically on the adaptive parameter bank 113. Referring now to
Fig. 3b, the adaptive parameter bank 113 includes one or more scaling vectors such as Y_R,
Y_I which are adapted by accumulating update signals u_r, u_i. The scaling vectors Y_R, Y_I are
applied to a tonal reference signal from lines 36 and 43, respectively, to generate the n
correction signals y_n. The parameter back-projection element 82 constrains adaptation of
20   scaling vectors Y_R, Y_I when unconstrained accumulation of update signals u_r, u_i would
cause one or more correction signals y_n to lie beyond a selected physical limit value
relating to a physical limitation of the system. The physical limit value would typically be
selected as a maximum allowable value of the means-squared voltage applied to the
respective actuator, or the maximum allowable value of means-squared current applied to
25   the respective actuator. In addition, it may be desirable that the physical limit value relate
to the maximum allowable value of the means-squared displacement for an output
component of the respective actuator, such as loudspeaker diaphragm displacement. This
maximum allowable value may be chosen in response to a peak amplitude limit in the case
of a tonal disturbance.

30   The controller 112 receives an input signal x(k) from an input sensor 30.
The input signal x(k) is transmitted to a phase-locked loop circuit 32 in the controller 112.
The phase-locked loop circuit 32 outputs a reference signal at a particular frequency,

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which is the frequency of the tone being attenuated. The reference signal is preferably a discrete time sequence in the form of a cosine wave at a particular frequency. It is preferred that the reference signal have a normalized magnitude (e.g. unity). Other methods of obtaining a reference signal can be used within the spirit of the invention,

5   however, the phase-locked loop circuit 32 is preferred because it enables frequency tracking and a normalized input signal. In most applications, it is preferred that the constraint surface S define a fixed surface in the parameter space for the adaptive parameters. However, in cases where reference signal statistics are non-stationary, it may be desirable to periodically redefine the constraint surface S in response to the changing

10   reference signal statistics. In the system shown in Fig. 3B, the reference signal $x(k)$ is generated by a phase-locked loop 32, so the use of a fixed constraint surface S is preferred.

The reference signal $x(k)$ is separated into two signals at junction 34: an in-phase reference signal is transmitted through line 36, and a quadrature reference signal

15   is transmitted through line 38. The in-phase reference signal is transmitted through line 36 to an in-phase scaling element 40. The in-phase scaling element 40 multiplies the in-phase reference signal by an in-phase scaling vector Y_R to generate n in-phase components y_r of the n correction signals y_n. The in-phase scaling element 40 stores the values of the in-phase scaling vector Y_R, and updates the values. The values of Y_R are updated by

20   summing the product of an in-phase update signal u_r multiplied by a step size μ, unless it is necessary to constrain adaptation so none of the correction signals y_n exceed the selected physical limit value.

Contemporaneously, quadrature components y_i of the correction signals y_n are generated. The quadrature reference signal is transmitted through line 38 to a phase

25   shifter 42 that shifts a quadrature reference signal 90° to in effect generate a sine wave corresponding to the cosine wave. The quadrature scaling element 44 multiplies the quadrature reference signal by a quadrature scaling vector Y_I to generate n quadrature components y_i of the n correction signals y_n. The scaling element 44 stores the values of the quadrature scaling vector Y_I and updates the values by summing the values of the

30   product of the quadrature update signal u_i multiplied by the step size μ, unless it is necessary to constrain adaptation so none of the correction signals y_n exceed the selected limit.

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The n in-phase output signals y_r and the n quadrature output signals y_i are summed at summer 46 to generate n correction signals y_n. The n correction signals y_n are transmitted to the n actuators 16.

The array of error sensors 22 generate p error signals e_p preferably every sample period k. The p error signals e_p are transmitted to error weighting element 126, which is similar to the error weighting element 26 in system 10 shown in Figs. 1a and 1b, however, it is preferred in system 110 that the in-phase weighting element 50 be represented by the Re $\{H_2\}$ and the quadrature weighting element 52 is represented by the Im $\{H_2\}$. The preferred parameter update generator 128 in system 110 shown in Figs. 3a and 3b is the same as the parameter update generator 28 preferably used in system 10 described in Figs. 1a and 1b.

Referring to Fig. 4, star 86 represents a point along the adaptation trajectory of the scaling vector Y_R, as the scaling vector Y_R is being adapted under fully normalized conditions, where the scaling vector Y_R traverses the constraint surface S. In the absence of the parameter back-projection element 82, normalized adaptation would attempt to occur from point 86 directly towards an optimum non-constrained solution 84 in accordance with the step size μ to point 88. For simple back-projection, adaptation beyond the constraint surface S is constrained by back-projecting from point 88 to the constraint surface S in a direction orthogonal to the constraint surface S to point 90. Performance of the system at point 90 is improved over the performance at point 86. In terms of error cost function, the point 90 is closer to the optimum non-constrained solution 84 than point 86. As the system continues to adapt and back-project to the constraint surface S, the constrained solution migrates along the constraint surface S to point 92. At point 92 along the constraint surface S, the direction of the unconstrained update vector u is approximately parallel to the direction of back-projection vector g, thus rendering point 92 as a final solution along the constraint surface S. However, the optimal constrained solution occurs at point 94 where the cost function performance curve is tangential to the constraint surface S. Therefore, it is desirable that constrained adaptation converge at point 94, rather than at point 92.

Referring now to Fig. 5, back-projected adaptation converges at the optimal constrained solution 94 if back-projection is compensated to account for adaptation normalization (i.e. compensated in accordance with the transformation matrix B). As long

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as the length of the back-projection is small and the rotation of back-projection does not exceed 90° with respect to the constraint surface S, the constraint surface S can be treated as a flat surface evaluated from the point of departure, and the back-projection will intersect the surface S.

5   Fig. 6 is a graphical depiction of back-projected adaptation which is compensated for normalized adaptation in accordance with the invention. In Fig. 6, vector $u = C^H e\mu$ represents the update signal using a gradient descent method. The vector $\chi = Bu$ represents a normalized update signal generated from the gradient descent vector via the transformation matrix B. The vector d_S is a vector normal to the constraint surface S

10   and the vector d_R is determined from d_S via the transformation matrix B according to the relation $d_S = Bd_R$. Compensated back-projection is illustrated by vector $-gd_R$. The normalized update vector lying tangent to the plane is shown in Fig. 6 as vector $\overline{\chi}$, where it is given by the vector sum $\overline{\chi} = \chi - gd_R$. The value for g is determined such that this vector sum lies tangent to the plane, or equivalently, such that it is orthogonal to d_S.

15   Using this method, the vector d_S sufficiently characterizes the constraint surface for the purpose of back-projection to a tangent plane. Fig. 5 illustrates that continued normalized adaptation with compensated back-projection results in the system converging at the optimum constrained solution 94.

          Fig. 7 illustrates the behavior of the back-projected update at the

20   intersection of multiple constraints. For instance, the intersection of the boundary of the constraint S_1 for a first actuator and the boundary of the constraint S_2 for a second actuator. Line 76 shows the trajectory of normalized adaptation towards the optimal unconstrained solution 84 until the scaling vector (i.e. adaptive parameters) reaches the selected limit S_2 for the second actuator. As adaptation continues in accordance with the

25   compensated back-projection technique, adaptation migrates from point 96 along the surface defined by S_2 to the intersection 98 between S_1 and S_2. However, at the intersection 98, the orientation of the tangent plane is not specifically defined. To overcome this problem, it is desirable to round the surface at the intersection 98 between constraint surfaces S_2 and S_1, see reference number 102. In this manner, back-projected

30   adaptation will progress around the rounded corner 102 until adaptation converges at the optimal constrained solution 94, located along S_1, as long as step size μ and/or transformation matrix B are selected properly.

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Fig. 7 graphically illustrates the use of a single constraint S to approximate multiple constraints surface S_1 and S_2. In mathematical terms, the preferred constraint function for a single tone system is defined as:

$$c(Y_R, Y_I) = \sum_n \left[\frac{(Y_{R,n}^2 + Y_{I,n}^2)}{G_n} \right]^p = 1 \qquad (2A)$$

And in a multiple tone system, the constraint function is defined as:

$$c(Y_R, Y_I) = \sum_n \left[\frac{\sum_t (Y_{R,n,t}^2 + Y_{I,n,t}^2)}{G_n} \right]^p = 1 \qquad (2A')$$

5   The constraint S is defined to be the set of points satisfying equations (2A) or (2A').

In equations (2A) and (2A'), Y_R and Y_I represent scaling vectors, G_n represents the maximum allowable output power level for the nth actuator, and p is a multiple constraint approximation factor. Choosing too small of a value of p can cause excessive and unnecessary power limiting. Using too large of a p value mandates the use

10   of a smaller step size μ. Hence, a trade-off exists between the level of approximation for multiple constraints and the adaptation rate which can be achieved.

If unconstrained adaptation causes one or more of the adaptive parameters to substantially lie outside of the constraint surface S, back-projection should be accomplished as follows. A vector normal to the constraint surface S can be found by

15   taking the gradient of $c(Y_R, Y_I)$ with respect to Y_R and Y_I. For a single tone case, a vector d_S normal to the constraint surface S is defined by:

$$d_S = [y_1(y^*_1 y_1)^{p-1}, ..., y_n(y_n^* y_n)^{p-1}]^T. \qquad (3A)$$

where operator * denotes taking the compar conjugate. Transforming the vector d_S by transformation matrix B results in:

20                                      $$d_R = Bd_S. \qquad (4A)$$

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Given an unconstrained update vector u, a back-projection gain factor g (scalar) is defined by the following equation:

$$g = \frac{\overline{\chi} d_S + d_S^H \overline{\chi}}{d_R^H d_S + d_S^H d_R}.$$ (5A)

The compensated, back-projected update $\overline{\chi}$ is defined by the following vector equation:

$$\overline{\chi} = Bu - gd_R.$$ (6A)

5   Due to the curvature of the surface S, a slight correction factor may be required such as:

$$y = g_c y \text{ where } g_c = (c(Y))^{-1/2p}$$ (7A)

Figs. 8a and 8b illustrate the performance of a multi-channel, normalized tonal adaptive control system based on compensated, back-projected adaptation to a smooth convex constraint surface S in which the p factor is chosen as 32, and the selected limit for the actuators is set at unity. In Fig. 8a, three curves 104, 106, 108 representing the

10   convergence for the sum of squared error signals are provided with respect to time. Curve 104 represents convergence for gradient descent adaptation with uncompensated back-projection, where the step size μ was chosen such that the convergence rate was maximized. Curve 106 represents convergence for normalized adaptation with uncompensated back-projection. Note that curve 106 converges more quickly than curve

15   104, however, curve 106 converges at an elevated level, e.g. star 92 in Figs. 4 and 5. Curve 108 represents convergence for normalized adaptation with compensated back-projection. Note that curve 108 converges as quickly as curve 106, however, continues to converge to a lower error signal value, e.g. star 94 in Figs. 4 and 5. In Fig. 8b, the magnitude of outputs for each of the 8 actuators is plotted with respect to time. For the

20   period of time from k = 0 to k = k_1, the system is adapting and it is not necessary to limit any of the actuators. At time k = approximately k_1, it is necessary to limit two of the actuators. Note that limiting two of the actuators causes overall system adaptation to adjust trajectories as illustrated by the changes in actuator output for several of the actuators at time k = approximately k_1. Between time k = k_1 and k = k_2, the system is

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adapting along the limit surface for the two actuators. At time k = approximately k_2, it is necessary to limit a third actuator, thus again creating some readjustment in the trajectory of some of the other actuators.

Fig. 9a illustrates another tonal embodiment of the invention including a regressor weighting element H_3, block 284. In many respects, the system shown in Fig. 9a is similar to the system shown in Fig. 3a and similar reference numerals are used where appropriate to facilitate understanding.

Referring to Fig. 9a, the system 210 includes an error weighting element H_2, block 226, and a regressor weighting element H_3, block 284. In order for the system 210 to be convergent, it is important that the weighting elements H_2 and H_3 be selected so that the eigenvalues of the product $H_3{}^H H_2 C$ have negative real part at the frequencies of interest. In order to account for delay or phase changes in the C path, the system 210 can be made more stable by providing delay or phase change by the regressor weighting element H_3, block 284. In such a system, H_3 is preferably set to a delay element of k_d samples. That is, its frequency response is given by H_3 equals $Ie^{-j\omega(kd/fs)}$, where ω is the radian frequency response of the disturbance and f_s is the sampling rate (number of samples per second) for the system 210. This delay or phase change term is useful for approximating the group delay or phase characteristics in the C path, and broadens the bandwidth of single frequency decompositions used in the C path model. Given the presence of the regressor weight element, the error weighting element is correspondingly set to $H_2 = -BC^H e^{-j\omega(kd/fs)}$ in order to account for the phase shift imposed by the regressor weighting element.

As shown in Fig. 9a, error input signals e from the error weighting element H_2, block 226, input the parameter update generator 228, as well as filtered regressor signals x'(k) from the regressor weighting element H_3, block 284. The parameter update generator 228 outputs update signals u which are used by the adaptive parameter bank 213 to update adaptive parameters. As discussed with respect to Fig. 1a, the adaptive parameter bank 13 generates a plurality of n correction signals y_n. Each of the n correction signals y_n drive the actuator 16 to provide cancelling secondary input 17 to the acoustic plant. When the system 210 is operating such that the n correction signals y_n do not exceed selected limits, the system 210 preferably operates in accordance with C path null space constraint techniques as described in U.S. Patent Application Serial No.

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08/369,925 entitled "Adaptive Tonal Control System With Constrained Output And Adaptation", by Steven R. Popovich, now U.S. Patent No. 5,633,795, issued on May 27, 1997. However, once it is determined that one of the correction signals y_n will exceed a selected limit, parameter back-projection, as illustrated by block 282 on block 229, is

5   desirable. The parameter back-projection element 282 shown in Fig. 9 is similar to the parameter back-projection element described with respect to Figs. 3a through Fig. 8.

Depending on the dimension of the systems 110 and 110a described with respect to Figs. 3a through Fig. 8, or the system 210 described with respect to Fig. 9a, matrix computations may become computationally burdensome, especially when the

10   system is operating to attenuate several distinct frequencies. One way to lessen computational burdens created by matrix multiplications both while implementing C path null space constraint techniques and during parameter back-projection is to accumulate the update signals u for a number of sample periods (e.g. 10-100 sample periods), combine the accumulated update with the respective adaptive parameter in the adaptive parameter bank,

15   and thereafter back-project the accumulated update to the constraint surface S, if necessary.

Referring to Fig. 9b, a time-sharing technique can be used in which processing requirements are reduced by selectively adapting with respect to the principle components of the system. For the system in Fig. 9b, the parameter update generator 228

20   and the error weighting element 226 shown in previous Figures is replaced by the combination of an error signal correlator/accumulator 228A and a time-sharing module 228B. The error signal correlator/accumulator 228A can be used to accumulate information relating to the phase and amplitude of the error signal according to the following equation:

25   $$\rho(k+1) = \rho(k) + e_p(k)[\chi_R'(k-k_d) + jx_I'(k-k_d)] \qquad (8A)$$

where $\rho(k)$ is a pxl complex vector representation for the accumulated error update signal, $e_p(k)$ is a pxl vector of error signals from the error sensors 22, $x_R'(k-k_d)$ is a delayed version of the in-phase regressor signal and $x_I'(k-k_d)$ is a delayed version of the quadrature reference signal, all at time k. The respective components of the accumulated error update

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signal $\rho(k)$ corresponding to columns of matrix $Ue^{-j\omega(kd/fs)}$ are determined in block 228B according to:

$$q_j = \tilde{U}_j^H \rho(k) \qquad (9A)$$

where q_j is the level of the component of $\tilde{U}_j$ present in the accumulated error update signal and $\tilde{U}_j$ denotes the jth column from matrix $Ue^{-j\omega(kd/fs)}$. The component $\tilde{U}_j$ is eliminated from

5   the accumulated update signal in block 228A, in accordance with the following equation:

$$\rho_{new} = \rho_{old} - q_j\tilde{U}_j \qquad (10A)$$

Since the columns of matrix U are orthogonal, they form a complete basis. Hence, as long as all components are periodically projected out of the accumulated error update signal, the accumulation represented by equation 8A remains bounded.

The update, and if necessary restraint, is then performed for each

10   component V_j corresponding to the respective $\tilde{U}_j$ and q_j. The component V is used to adapt the adaptive parameters in block 213 according to the following equation:

$$Y_{R,new} = Y_{R,old} + s_jq_j \, Re\{V_j\} \text{ and } Y_{I,new} = Y_{I,old} + s_jq_j \, Im\{V_j\} \qquad (11A)$$

where s_j represents a normalization factor determined in accordance with the magnitude of the corresponding singular value from the decomposition of the C path model. If the

15   adaptive parameters lie with the constraint surface S, the component V_j is used to adapt the adaptive parameters in accordance with null space restraint techniques (i.e. the values for s_j corresponding to trivial or zero singular values are set to zero). If the adaptive parameters would substantially lie outside of the constraint surface S (i.e. substantially beyond the tangent plane) after adaptation, the component V_j is used to adapt the adaptive

20   parameters in accordance with the back-projection techniques, as described earlier. In particular, the adaptation is carried out according to:

$$Y_{R,new} = Y_{R,old} + s_jq_j \, Re\{\overline{V}_j\} \text{ and } Y_{I,new} = Y_{I,old} + s_jq_j \, Im\{\overline{V}_j\} \qquad (12A)$$

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where $\overline{V}_j$ is a back-projected version of V_j. These back-projected versions can be periodically updated as the adaptive parameters migrate along the constraint surface. Adaptation can occur with respect to any number of columns in V as long as each column in V is processed within a reasonable time frame. Such a time-sharing method reduces or

5   eliminates the need for complete matrix multiplications, and thus allows for compensated and back-projected adaptation when using a DSP having conventional processing capabilities.

Broadband Control System

Fig. 10 shows an active adaptive attenuation system 310 as disclosed in

10   issued U.S. Patent No. 5,621,803 entitled "Active Attenuation System With On-Line Modeling of Feedback Path", to Trevor Laak, issued on April 15, 1997 which is herein incorporated by reference and is assigned to the assignee of the present application. The system 310 includes an actuator 311 that outputs a secondary input that combines with a system input 312 to yield a system output 314. The system 310 shown in Fig. 10 is a

15   feedforward system, and is capable of attenuating or shaping acoustic disturbances in the system input 312 that are not periodic. (The system 310 is also capable of attenuating or shaping tonal disturbances.) The system includes an input sensor 16, such as a microphone or accelerometer, which senses the system input 312 and generates an input signal that is transmitted from the sensor 316 through line 318. An error sensor 320

20   senses the system output 314 and generates an error signal which is transmitted through line 322. The system 310 uses an adapter controller 321, preferably embodied in a digital signal processor to drive the actuator 311. A first adaptive filter model 324, block A, in the adaptive controller 321 has a model input from line 319 derived from the input signal in line 318, an error input from line 321 derived from the error signal in line 322, and a

25   model output which is a correction signal that is transmitted through line 326 to the actuator 311, as is known in the art.

The transfer function of the C path from the output of the A model 324 to the output of the error sensor 320 is modeled by another adaptive filter model 328, block C, preferably as disclosed in U.S. Patent No. 4,677,676. The C model has a model

30   input from an auxiliary random noise source 330, block N, which provides random noise uncorrelated with the system input 312. The output of C model 328 is subtracted at summer 332 from the error signal 322, and the resultant sum is multiplied at multiplier

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334 with the input to the C model 328. The multiplier 334 outputs a weight update signal in line 335 for the C model 328. The random noise signal from source 330 is also summed at summer 336 with the correction signal from A model 324, and the resultant sum is transmitted to the actuator 311. A copy 338 of the C model receives input from

5   line 319 which is the same input that inputs the first adaptive filter model 324, block A. The C model copy 338 outputs a filtered regressor signal which is transmitted through line 339 to adaptive parameter generator 340 (e.g. multiplier 340). The multiplier 340 multiplies the error signal from line 322 and the filtered regressor signal from line 339, and outputs an update signal in line 321 that is used to update the first adaptive filter

10   model 324, block A.

       A second adaptive filter model 342, block D, receives model input from the summer 336 through line 343, receives error input from multiplier 350 through line 351, and outputs a recursive signal in line 353 that is transmitted to summer 344. The recursive signal in line 353 is summed with the input signal in line 318 by summer 344 to

15   generate the reference signal in line 319 which is supplied to the first adaptive filter model 324, block A. The error input signal for the D model 342 in line 351 is generated in multiplier 350 by multiplying the error signal in line 322 by a filtered correction signal in line 343. The correction signal in line 343 is filtered by a copy 346 of the A model 324, and a copy 348 of the C model 328 both in series. The purpose of the D model is to

20   model the acoustic feedback path between the actuator 311 and the input sensor 316 on-line, and electrically remove the effect of acoustic feedback from the reference signal in line 319. Preferably, both the A model 324 and the D model 342 are FIR (finite impulse response) filters implemented in the time domain, and updated using a normalized gradient descent method such as the LMS (lease means square) or RLMS (recursive lease means

25   square) techniques shown in Fig. 10.

       Fig. 11 shows the adaptive control system 310 implementing a parameter back-projection element 352 to constrain adaptation in accordance with the invention. The purpose of the parameter back-projection element 352 is to constrain adaptation of adaptive parameters in the A model 324 so that no correction signal in line 326 exceeds a

30   selected limit S. While the invention can be carried out in a system 310 implementing only an FIR A model without a recursive model such as a D model 342, or a B model as disclosed in U.S. Patent No. 4,677,676, it is preferred that the system 310 implement a D

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model 342 to help maintain the statistics of the reference signal 319 stationary or nearly stationary. If reference signal statistics are nearly stationary, a fixed constraint surface S in the parameter space can be used, otherwise it may be desirable to select the constraint surface S in terms of reference signal statistics.

Fig. 12 illustrates a constraint surface 354, S, defined in the parameter space for the adaptive parameters in relation to an error performance contour map for two adaptive parameters a_1 and a_2. The optimum non-constrained solution is depicted by star 356. The optimum constrained solution is depicted by star 358 which is located on the constraint surface 354 at the location where the constraint surface 354 is tangent to one of the error contours for the performance map. The constraint surface 354 in the parameter space for the adaptive parameters is typically elliptical because the surface 354 will typically represent a constraint limit related to the means square value of current, voltage, or displacement for the actuator 311.

Fig. 13 is a graphical depiction of compensated, back-projected adaptation for the broadband system 310 shown in Fig. 11. In Fig. 13, vector d_S is a vector normal to the constraint surface c(a). Vector d_R is determined from d_S via the transformation matrix B according to the relation $d_S = Bd_R$. The vector $u = [Cx]e\mu$ represents the unconstrained update signal vector using a gradient descent method. The vector $\chi = Bu$ represents a normalized update signal generated from the gradient descent vector via the transformation matrix B. Compensated back-projection is illustrated by vector $-gd_R$. The normalized update vector lying tangent to the plane is shown in Fig. 13 as vector $\overline{\chi}$, where it is given by the vector sum $\overline{\chi} = \chi - gd_R$. The value for g is determined such that this vector sum lies tangent to the plane, or equivalently, such that it is orthogonal to d_S. Continued adaptation as illustrated in Fig. 13 results in the system converging at the optimum constrained solution indicated by star 358 in Fig. 12.

The preferred manner of carrying out compensated, back-projected adaptation for a single input single output (SISO) broadband system 310 as shown in Fig. 11 is explained in mathematical terms as follows.

The transformation matrix B is preferably determined by taking the eigenvalue decomposition of the autocorrelation matrix:

$$R_{X_cX_c} = V\Lambda V^H \qquad (1B)$$

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where V is a square matrix, V^H is the hermitian transpose of matrix V, and Λ is a matrix containing eigenvalues of the system along the diagonal. The off-diagonal elements of Λ are 0 while the diagonal elements are in general real and positive. The transformation matrix B is preferably calculated as $B = V\overline{\Lambda}V^H$, where $\overline{\Lambda}$ is determined by inverting non-trivial values on the diagonal of Λ down to an inversion limit defined in relation to the maximum eigenvalue.

The unconstrained update signal u in line 321, Fig. 11, before normalization is represented by $u = [Cx] e\mu$ where $[Cx]$ is the filtered reference signal regressor, line 339, Fig. 11, e is the error signal in line 322, Fig. 11, and μ is a convergence step size. The normalized unconstrained update signal vector χ is given by $\chi = Bu$.

The constraint surface S for a single input single output system 310 having a single constraint is defined as the set of all points satisfying:

$$c(a) = \frac{a^T R_{KK} a}{G} = 1 \qquad (2B)$$

where R_{KK} is a non-identity covariance matrix for the term K(k) which represents the convolution between the reference signal x(k) and the transfer function H(k) of the path which translates the correction signal y(k) into a physical limit value relating to the physical limitations of the system; a is the tap weight vector for the first adaptive filter 324, block A (i.e. the adaptive parameters); and G represents the maximum allowable means-squared output (e.g. power) for the actuator 311. If applying the normalized, unconstrained update signal vector χ to the adaptive parameters, a, does not cause the adaptive parameters to lie outside of the constraint surface S, normalized adaptation proceeds unconstrained. However, if applying the normalized, unconstrained update signal vector χ to the adaptive parameters a results in adaptive parameters substantially outside of the constraint surface S, back-projection is used to adapt the adaptive parameters along the constraint surface S.

Back-projection is explained mathematically as follows. A vector d_s which is normal to the constraint surface S at a point on S is determined by a scaled version of the gradient for the constraint function c(a) evaluated at that point, as represented by:

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$$d_S = R_{KK}a. \qquad (3B)$$

Transforming the vector d_S by the transformation matrix B results in:

$$d_R = B\ d_S. \qquad (4B)$$

Given a normalized, unconstrained update vector $\chi = B\ u$, a back-projection gain factor g
(scaler) is defined by the following equation:

$$g = \frac{d_S^T u}{d_R^T d_S}. \qquad (5B)$$

The normalized, constrained update signal vector χ is defined by the following vector
equation:

$$\overline{\chi} = u - gd_R \qquad (6B)$$

Applying the normalized, constrained update signal vector χ to the respective adaptive
parameters a results in adaptation along the constraint surface S.

In some applications, it may be desirable to provide two or more separate
constraints on the adaptive parameters contemporaneously. Fig. 15 illustrates an
application involving two separate constraints. It is desirable to combine the constraint
functions to provide a single smooth constraint surface for back-projection. In Fig. 15, a
first constraint function 366 is illustrated in the parameter space of the adaptive parameters
a_0 and a_1. A second constraint function 368 is also shown in the adaptive parameter space
for the adaptive parameters a_0 and a_1. For purposes of illustration, the first constraint
function can be represented by $c_1(a) = (a^T R_{KK,1}\ a) \div G_1 = 1$ and the second constraint
function 368 can be defined as $c_2(a) = (a^T R_{KK,2}\ a) \div G_2 = 1$. In order that adaptation does
not generate one or more adaptive parameters lying substantially outside of either of the
constraint functions 366 or 368, a constraint surface 370 representing a combination of
each individual constraint 366 and 368 is used to constrain adaptation. Note that the
portions of the combined constraint surface 370 corresponding to the intersections 372 of
the first and second constraint functions 366 and 368 should be smooth to ensure stability.

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In general, the constraint surface for a system having multiple constraints is preferably defined by the following equation:

$$c(a) = \sum_{m=1}^{M} \left(\frac{(a^H R_{KK,m} a)}{G_m} \right)^p \qquad (2B')$$

In such a system, a vector d_S normal to the constraint surface S is again defined by a scaled version of the gradient for the constraint function $c(a)$ according to:

$$d_S = \sum_{m=1}^{M} G_m^{-P} \left(a^H R_{KK,m} a \right)^{P-1} R_{KK,m} a \qquad (3B')$$

5   While the system 310 shown in Fig. 11 has been described thus far as a single input single output (SISO) system, it should be apparent to those skilled in the art that such a system could include multiple actuators 311 and multiple microphones 320 (i.e. a MIMO multiple input multiple output system). In a MIMO system, or even in some SISO systems, computational burdens created by matrix multiplications may render it

10   desirable to accumulate unconstrained update signals χ for a number of sample periods (e.g. 10 to 100 sample periods), combine the accumulated update with the respective adaptive parameter in the adaptive parameter bank, and thereafter back-project the accumulated update to the constraint surface S, if necessary. Alternatively, it may be desirable to adapt via time-sharing among linearly independent coordinates of the system

15   in a manner similar to the description of time-sharing for the tonal system 110, Figs. 3a and 3b. In particular, updates are accumulated for each sampling period in accordance with the following equation:

$$\chi_{acc}(k+1) = \chi_{acc}(k) + x_c(k) \, e(k) \, \mu \qquad (8B)$$

where $\chi_{acc}(k)$ is the accumulated update at time k, e(k) is the error signal in line 322, Fig.

20   11 at time k, $x_c(k)$ is the filtered regressor signal in line 339, Fig. 11, at time k, and μ is a convergence step size. The respective components q of the accumulated update signal $\chi_{acc}(k)$ corresponding to the respective columns of matrix V are given by:

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$$q = V_i^H \chi(k) \qquad (9B)$$

where q is the level of the accumulated update signal $\chi_{acc}(k)$ in the direction of V_i. The component q is then eliminated from the accumulated update signal $\chi_{acc}(k)$ in accordance with the following equation:

5

$$\chi(k+1) = \chi_{acc}(k) - q\ V_i \qquad (10B)$$

Since the columns of matrix V form a complete basis, periodically projecting out respective components does not render the system unstable. The update is then performed for each component V_i in accordance with the following expression:

$$A(k+1) = A(k) + \frac{q}{\Lambda_{ii} V_{i,\ constrained}} \qquad (11B)$$

where Λ_{ii} is the corresponding diagonal element in the Λ matrix, and $V_{i,\ constrained}$ is the

10   projection of the ith column of the V matrix onto the constraint surface S.

The invention has been described with respect to a few preferred embodiments of the invention. Various alternatives, modifications and equivalents may be apparent to those skilled in the art. The following claims should be interpreted to include such alternatives, modifications and equivalents.

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CLAIMS

I claim:

1. An adaptive tonal control system having a system input and a system output, the adaptive tonal control system comprising:

a plurality of actuators each receiving a correction signal and outputting a secondary input, the secondary input combining with the system input to yield the system
5   output;

a plurality of error sensors sensing a system output, each error sensor generating an error signal; and

an adaptive controller that outputs the correction signals, the controller including

10   an adaptive parameter bank that outputs a plurality of output

signals in accordance with the adaptive parameters, the output signals

being used to generate the correction signals;

a parameter update generator that generates update signals in

accordance with the error signals to adapt the adaptive parameters in the

15   adaptive parameter bank; and

a parameter back-projection element that directly limits adaptation

of the adaptive parameters so that none of the correction signals drive the

respective actuator beyond a constraint surface which is defined in the

parameter space of the adaptive parameters.

2.   An adaptive tonal control system as recited in claim 1 wherein a reference signal inputs the adaptive parameter bank and the constraint surface is defined in the parameter space of the adaptive parameters as a function of reference signal characteristics.

3.   An adaptive tonal control system as recited in claim 1 wherein a reference signal inputs the adaptive parameter bank and the constraint surface is fixed in the parameter space for the adaptive parameters.

4.   An adaptive tonal control system as recited in claim 1 wherein:

the parameter back-projection element generates back-projection signals which are combined with update signals so that adaptation of the adaptive parameters in the adaptive parameter bank are constrained in accordance with the constraint surface.

5.   An adaptive tonal control system as recited in claim 1 further comprising:

a C model of a path between the output of the adaptive controller and the error sensors; and

an error weighting element that inputs the error signals from the error sensors and weights the error signals to generate error input signals that are input to the parameter update generator, the error weighting element including a matrix representing BC^H, where B is a transformation matrix and C^H is the hermitian transpose of a matrix C representing the C model.

6.   An adaptive tonal control system as recited in claim 5 wherein B is an n x n transformation matrix defined by $B = V\bar{\Lambda}V^H$ where matrix V is determined in accordance with an eigenvalue decomposition of C^HC, V is an n x n unitary matrix, V^H is the hermitian transpose of V, $\bar{\Lambda}$ is a real diagonal matrix containing the eigenvalues of C^HC, and $\bar{\Lambda}$ is formed by inverting non-trivial diagonal entries of Λ down to an inversion limit defined in relation to the maximum eigenvalue.

7.   An adaptive tonal control system as recited in claim 1 wherein the constraint surface is a smooth convex surface.

8.   An adaptive tonal control system as recited in claim 1 in which the constraint surface is defined by actuator-specific output limitations.

9.   An adaptive tonal control system as recited in claim 1 wherein the system includes n actuators, the adaptive parameters in the adaptive parameter bank include a set of in-phase scaling vectors $Y_{R,n}$ for the n actuators and a set of quadrature

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scaling vectors $Y_{I,n}$ for the n actuators, and the selected limit is defined by a constraint surface $c(Y_R, Y_I)$ such that:

$$c(Y_R, Y_I) = \sum_n \left[\frac{Y_{R,n}^2 + Y_{I,n}^2}{G_n} \right]^p = constant$$

where G_n is a gain factor for the nth actuator, and p is a multiple constraint approximation factor.

10.   An adaptive tonal control system as recited in claim 1 wherein the system is a multi-tone adaptive control system, the system includes n actuators, the adaptive parameters in the adaptive parameter bank include a set of in-phase scaling vectors $Y_{R,n,t}$ for the n actuators at each respective tone t and a set of quadrature scaling vectors $Y_{I,n,t}$ for the n actuators at each respective tone t, and the selected limit is defined by a constraint surface $c(Y_R, Y_I)$ such that:

$$c(Y_R, Y_I) = \sum_n \left[\frac{\sum_t (Y_{R,n,t}^2 + Y_{I,n,t}^2)}{G_n} \right]^p = constant$$

where G_n is a gain factor for the nth actuator, and p is a multiple constraint approximation factor.

11.   An adaptive tonal control system as recited in claim 1 further comprising a regressor weighting element that receives an input reference signal and outputs a filtered regressor signal that inputs the parameter update generator.

12.   An adaptive tonal control system as recited in claim 11 wherein the adaptive controller further comprises:

a C model path between the output of adaptive controller and the error sensors; and

wherein the error weighting element is represented by $H_2 = -BC^H e^{-j\omega(kd/f_s)}$, and the regressor weighting element is represented by $H_3 = I e^{-j\omega(kd/f_s)}$ where ω is the

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frequency of the tone of interest, k_d is the amount of desired delay, f_s is the system
sampling rate, and transformation matrix $B = V\overline{\Lambda}V^H$ where matrix V is determined in
accordance with an eigenvalue decomposition of $C^H C$, V is a unitary matrix, B^H is the
hermitian transpose of matrix V, Λ is a real diagonal matrix containing the eigenvalues of
$C^H C$, and $\overline{\Lambda}$ is formed by inverting non-trivial diagonal entries of Λ down to an inversion
limit defined in relation to the maximum eigenvalue.

13.   An adaptive tonal control system as recited in claim 1 wherein
adaptation is accomplished via time-sharing.

14.   An adaptive tonal control system as recited in claim 13 wherein
time-sharing is accomplished by accumulating parameter updates, extracting components of
the accumulated updates in accordance with principal components of a C matrix modelling
the speaker-error path, and performing respective updates in accordance with the respective
component of the accumulated update.

15.   An adaptive tonal control system as recited in claim 14 wherein:
        the principal components extracted from the accumulated updates are
defined by columns of U and the respective components added to the adaptive parameters
are defined by the columns of matrix V, U and V being defined from singular value
decomposition of the C matrix.

16.   An adaptive tonal control system as recited in claim 15 wherein
each respective update is constrained by calculating the respective update in accordance
with a back-projected version of the respective column of matrix V.

17.   In an adaptive control system having a system input and a system
output, a method of controlling a tonal disturbance in the system output comprising the
steps of:
        filtering a reference signal through adaptive parameters to generate a
plurality of correction signals;

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driving a plurality of actuators in accordance with the correction signals
to generate a plurality of secondary inputs that combine with the system input to yield the
system output;

sensing the system output and generating a plurality of error signals in
10   response thereto;

using the error signals to generate a unconstrained update signal vector
that is intended to be used to adapt the adaptive parameters; and

constraining adaptation of the adaptive parameters in relation to a smooth
convex constraint surface surrounding a desired region in the parameter space of the
15   adaptive parameters.

18.   The method as recited in claim 17 wherein adaptation is
constrained by back-projecting the update signal vector onto or near the constraint surface
surrounding the desired region in the parameter space of the adaptive parameters when
using the unconstrained update signal vector for adaptation would cause one or more
5   adaptive parameters to lie outside of the desired region.

19.   The method as recited in claim 17 wherein the constraint surface
of the desired region is defined in the parameter space of the adaptive parameters as a
function of reference signal characteristics.

20.   The method as recited in claim 27 wherein the constraint surface
of the desired region is fixed in the parameter space of the adaptive parameter.

21.   The method as recited in claim 17 wherein:

the unconstrained update vector is generated using the error signals in
accordance with a gradient descent method to reduce a cost function; and

adaptation is constrained by vector summing a back-projection vector with
5   the unconstrained update signal vector so that none of the adaptive parameters lie
substantially outside of the constraint surface in the parameter space.

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22. The method as recited in claim 21 wherein the back-projection vector is orthogonal to the constraint surface in the parameter space.

23. The method as recited in claim 21 wherein the combination of the back-projection vector with the unconstrained update signal vector results in adaptation along a plane that is tangent to the constraint surface at the point which unconstrained adaptation would traverse the constraint surface.

24. The method as recited in claim 21 further comprising the steps of:

compensating the unconstrained update signal vector to normalize adaptation; and

compensating the back-projection vector to account for the compensation of the unconstrained update signal vector; and

wherein vector summing of the back-projection vector with the unconstrained update signal vector is accomplished by vector summing the compensated back-projection vector to the compensated unconstrained update signal vector.

25. The method as recited in claim 24 wherein the unconstrained update signal vector is compensated by transforming the unconstrained update vector by transformation matrix B, and the back-projection vector is compensated by transforming the back-projection vector by the same transformation matrix B and scaling so that the constrained update signal vector does not lie substantially outside of the constraint surface.

26. The method as recited in claim 25 wherein the transformation matrix B is a positive semi-definite matrix.

27. The method as recited in claim 17 wherein the smooth convex surface is defined by the following equation:

$$c(Y_R, Y_I) = \sum_n \left[\frac{Y_{R,n}^2 + Y_{I,n}^2}{G_n} \right]^p = constant$$

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where $Y_{R,n}$ represents the in-phase scaling vector for the nth actuator, $Y_{I,n}$ represents the quadrature scaling vectors for the nth actuator, p is a multiple constraint approximation factor, and G_n is a gain factor for the nth actuator.

28.   The method as recited in claim 17 wherein a plurality of t tonal disturbances are controlled by the method wherein the smooth convex surface is defined by the following equation:

$$c(Y_R, Y_I) = \sum_n \left[\frac{\sum_t (Y^2_{R,n,t} + Y^2_{I,n,t})}{G_n} \right]^p = constant$$

where n represents the number of actuators, t represents the number of tones being cancelled, $Y_{R,n,t}$ represents the in-phase scaling vector for the nth actuator for the respective tone t, $Y_{I,n,t}$ represents the quadrature scaling vector for the nth actuator for the respective tone t, p is a multiple constraint approximation factor, and G_n is a gain factor for the nth actuator.

29.   A method as recited in claim 17 wherein:

the unconstrained update signal vectors are accumulated over a plurality of sample periods; and

constrained adaptation of the adaptive parameters is accomplished via a time-sharing procedure in which orthogonal components of the accumulated update signal vectors are extracted individually from the accumulated update signal vector and the respective orthogonal component after being back-projected is used for constrained adaptation of the adaptive parameters.

30.   In an adaptive control system having a system input and a system output, a method of attenuating a disturbance comprising the steps of:

filtering a reference signal through adaptive parameters to generate a correction signal;

driving an actuator in accordance with the correction signal to generate a secondary output which is combined with a secondary input to yield the system output;

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sensing the system output and generating an error signal in response thereto;

using an error signal to generate a pre-constrained update signal vector that is intended to be used to adapt the adaptive parameter; and

constraining the adaptive parameters to lie within a constraint surface defined by the following expression:

$$c(a) = \frac{a^H R_{KK} a}{G} = constant$$

where R_{KK} is a non-identity covariance matrix for K(k) which represents the convolution between the reference signal and the transfer function H(k) of the path which translates the correction signal into a physical limit value relating to physical limitations of the system, a represents the adaptive parameters, and G represents the maximum allowable gain for the actuator.

31. A method as recited in claim 29, wherein the transfer function H(k) is equal to 1.

32. A method as recited in claim 30 wherein the transfer function H(k) is equal to 1.

33. A method as recited in claim 30 wherein the physical limit value represents the maximum allowable value of means-squared voltage applied to the actuator.

34. A method as recited in claim 30 wherein the physical limit value relates to the maximum allowable value of means-squared current applied to the actuator.

35. A method as recited in claim 30 wherein the physical limit value represents the maximum allowable value of means-squared displacement for an output element of the actuator.

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36.   In an adaptive control system having a system input and a system output, a method of attenuating a disturbance comprising the steps of:

  filtering a reference signal through adaptive parameters to generate a correction signal;

5   driving an actuator in accordance with the correction signal to generate a secondary output which is combined with the system input to yield the system output;

  sensing the system output and generating an error signal in response thereto;

  using the error signal to generate an unconstrained update signal vector

10 that is intended to be used to adapt the adaptive parameters; and

  constraining adaptation of the adaptive parameters to lie within or near a smooth convex constraint surface surrounding a desired region in the parameter space of the adaptive parameter satisfying two or more physical limitations of the system.

37.   A method as recited in claim 36 wherein one of the physical limitations of the system relates to maximum displacement of an output element of the actuator.

38.   A method as recited in claim 36 wherein one of the physical limitations of the system relates to voltage applied to the actuator.

39.   A method as recited in claim 36 wherein one of the physical limitations of the system relates to current applied to the actuator.

40.   A method as recited in claim 36 wherein the adaptive parameters are used to generate a plurality of correction signals, the plurality of correction signals drive a plurality of actuators each generating a secondary input that combines with the system input to yield the system output, and the system output is sensed to generate a

5 plurality of error signals each being used in turn to generate one of a plurality of unconstrained update signal vectors each intended to be used to adapt the adaptive parameters.

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41.   A method as recited in claim 36 wherein the smooth convex constraint surface is defined by the following expression:

$$c(a) = \sum_{m=1}^{M} \left(\frac{(a^H R_{KK,m} a)}{G_m} \right)^p \qquad (2B')$$

where m is the number of physical limitations on the system, $R_{KK,m}$ is a non-identity covariance matrix for the term K(k) which represents a convolution between each respective reference signal and the transfer function H(k) of the path which translates the respective correction signal into a physical limit value relating to the physical limitations of the system, a represents the respective adaptive parameter, and G_m represents the maximum allowable gain for the mth actuator.

42.   A method as recited in claim 36 wherein constrained adaptation is accomplished by back-projecting the unconstrained update signal vector in relation to the constraint surface surrounding the desired region in the parameter space of the adaptive parameters when using the unconstrained update signal vector causes the adaptive parameters to lie outside of the desired region in the parameter space of the adaptive parameters.

43.   A method as recited in claim 36 wherein adaptation of the adaptive parameters is constrained intermittently.

44.   A method as recited in claim 36 wherein a plurality of unconstrained update signal vectors are combined for adaptation prior to constraining adaptation for the respective adaptive parameter so that the adaptive parameter does not lie substantially outside of the desired region in the parameter space.

45.   A method as recited in claim 42 wherein back-projecting the unconstrained update signal vector is accomplished by back-projecting the unconstrained update signal vector to a plane that is tangent to the constraint surface at a point in which unconstrained adaptation would traverse the constraint surface.

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46.   A method as recited in claim 36 wherein:

the unconstrained update signal vector generated using the error signal is generated in accordance with a gradient descent method to reduce the cost function; and

adaptation is constrained by vector summing a back-projection vector with

5   the unconstrained update signal vector so that none of the adaptive parameters lie substantially outside of the smooth convex surface in the parameter space.

47.   The method as recited in claim 46 further comprising the steps of:

compensating the unconstrained update signal vector to normalize

adaptation; and

compensating the back-projection vector to account for the compensation

5   of the unconstrained update signal vector; and

wherein vector summing of the back-projection vector with the unconstrained update signal vector is accomplished by vector summing the compensated back-projection vector to the compensated unconstrained update signal vector.

48.   A method as recited in claim 47 wherein the unconstrained update signal vector is compensated by transforming the unconstrained update vector in accordance with transformation matrix B, and the back-projection vector is compensated by transforming the back-projection vector by the same transformation matrix B and

5   scaling so that the constrained update signal vector does not lie substantially outside of the constraint surface.

49.   A method as recited in claim 48 wherein the transformation matrix B is positive and semi-definite.

50.   A method as recited in claim 42 wherein:

the unconstrained update signal vectors are accumulated over a plurality of sample periods; and

constrained adaptation of the adaptive parameters is accomplished via a

5   time-sharing procedure in which linearly independent components of the accumulated update signal vectors are extracted individually from the accumulated update signal vector

and the respective linearly independent component after being back-projected is used for constrained adaptation of the adaptive parameters.

51.   A method as recited in claim 50 wherein the linearly independent components are orthogonal components.

52.   In an adaptive control system capable of attenuating non-repetitive acoustic disturbances and having a system input and a system output, a method of attenuating a non-repetitive acoustic disturbance comprising the steps of:

filtering a reference signal through adaptive parameters to generate a

5   plurality of correction signals;

driving a plurality of actuators in accordance with the correction signals to generate a plurality of secondary inputs that combine with the system input to yield the system output;

sensing the system output and generating a plurality of error signals in

10   response thereto;

using the error signals to generate an unconstrained update signal vector that is intended to be used to adapt the adaptive parameters; and

constraining adaptation of the adaptive parameters to lie substantially within or near a desired region in the parameter space of the adaptive parameters enclosed

15   by a smooth surface characterizing physical limitations of the system.

53.   A method as recited in claim 52 wherein adaptation of the adaptive parameters is constrained intermittently.

54.   A method as recited in claim 52 wherein a plurality of unconstrained update signal vectors are combined for adaptation prior to constraining adaptation for the respective adaptive parameter so that the adaptive parameter does not lie substantially outside of the desired region in the parameter space.

55.   An active acoustic attenuation system for attenuating a non-repetitive acoustic disturbance, the system comprising:

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an adaptive filter model including a set of adaptive parameters, the adaptive filter model inputting a reference signal and outputting a correcting signal;

5   an actuator that inputs the correction signal and outputs a secondary input that combines with the acoustic disturbance to attenuate or shape the acoustic disturbance;

an error sensor that senses system performance and generates an error signal in response thereto, the error signal being used to adapt the adaptive parameters in the adaptive filter model;

10   wherein adaptation of the adaptive parameters is constrained so that the adaptive parameters lie substantially within or near a desired region in the parameter space of the adaptive parameters enclosed by a surface in the parameter space characterizing one or more physical limitations of the system.

56.   A system as recited in claim 55 wherein the adaptive filter model is an FIR filter.

57.   A system as recited in claim 55 wherein the adaptive filter model is a recursive IIR filter.

58.   A system as recited in claim 55 wherein the recited adaptive filter model is a first adaptive filter model and the system further comprises a second adaptive filter model including a set of adaptive parameters, the second adaptive filter model inputting the correction signal and outputting a recursive signal that is combined with a

5   system input signal to generate the reference signal that inputs the first adaptive filter model.

59.   A system as recited in claim 55 wherein constrained adaptation is accomplished by back-projection means, said back-projection means constraining adaptation of the adaptive parameters when unconstrained adaptation causes one or more of the adaptive parameters to lie outside of the desired region in the parameter space of the

5   adaptive parameters.

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60.   A system as recited in claim 59 further comprising compensation means for compensating the unconstrained update signal vector to accomplish normalized adaptation and back-projection.

61.   A system as recited in claim 55 wherein adaptation of the adaptive parameters is constrained intermittently.

62.   A system as recited in claim 59 wherein a plurality of adaptive parameter update signal vectors are combined prior to back-projection to or near a constraint surface surrounding the desired region in the parameter space of the adaptive parameters.

63.   A system as recited in claim 55 wherein the system comprises a plurality of actuators and a plurality of error sensors, and the adaptive filter model includes a plurality of adaptive filter channels each generating a correction signal for a respective actuator.

64.   A method of adaptive control comprising the steps of:
       filtering a reference signal through adaptive parameters to generate a correction signal;
       driving an actuator in accordance with the correction signal to generate a secondary input that combines with a system input to yield a system output;
       sensing the system output and generating an error signal in response thereto;
       using the error signal to generate an unconstrained update signal vector that is intended to be used to adapt the adaptive parameters;
       constraining adaptation in accordance with physical limitations of the system by back-projection of the unconstrained update signal vector onto a plane calculated to lie tangent to a surface surrounding a desired region in the parameter space of the adaptive parameters; and

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periodically scaling the adaptive parameters to account for curvature of

15   the surface surrounding the desired region so that adaptive parameters do not lie outside of

the desired region.

65.   A method as recited in claim 64 further comprising the step of:

periodically correcting the orientation of the plane due to migration along

the surface surrounding the desired region in the parameter space of the adaptive

parameters.

FIG. 1a PRIOR ART

FIG. 1b
PRIOR ART

FIG. 2

FIG. 3a

5/15

110a

FIG. 3b

## FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8a

FIG. 8b

FIG. 9a

FIG. 9b

FIG. 10
PRIOR ART

FIG. 11

14/15
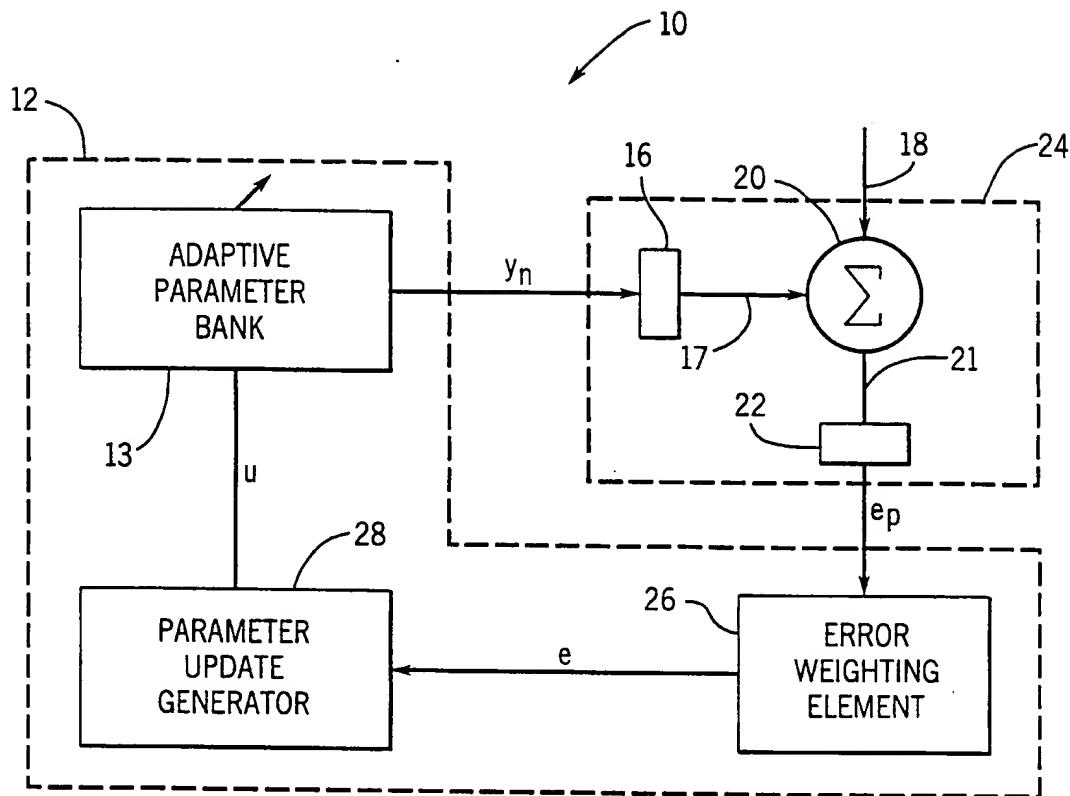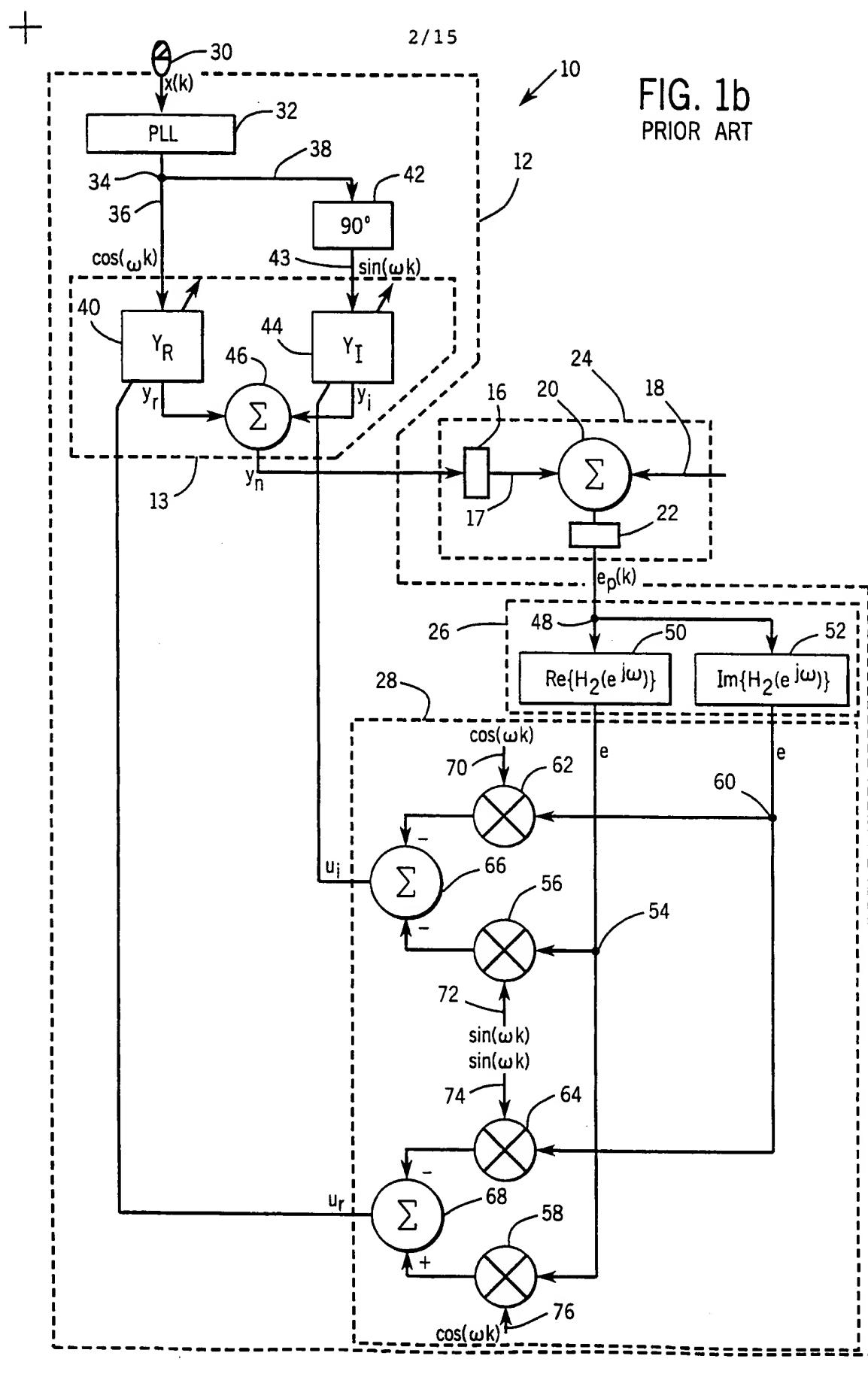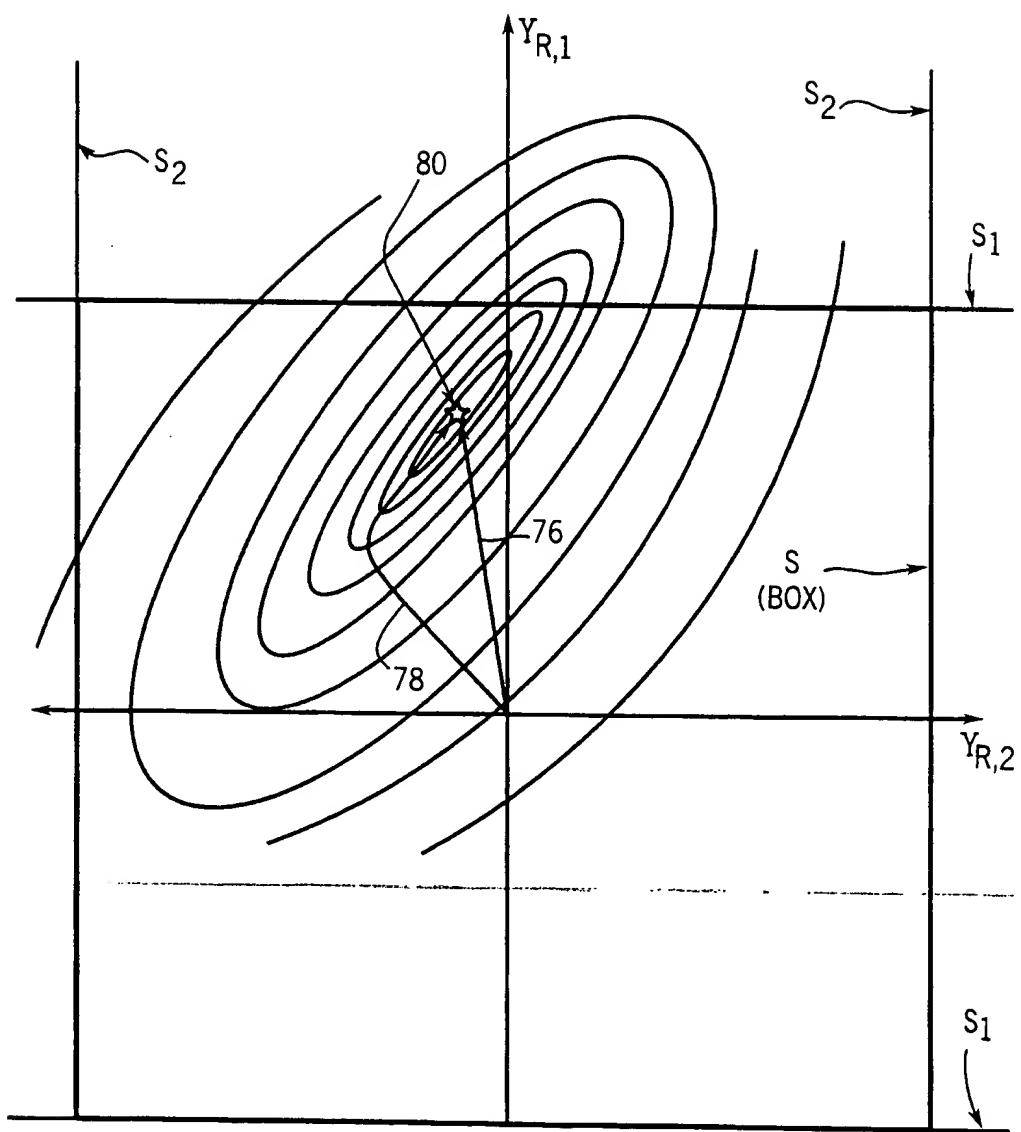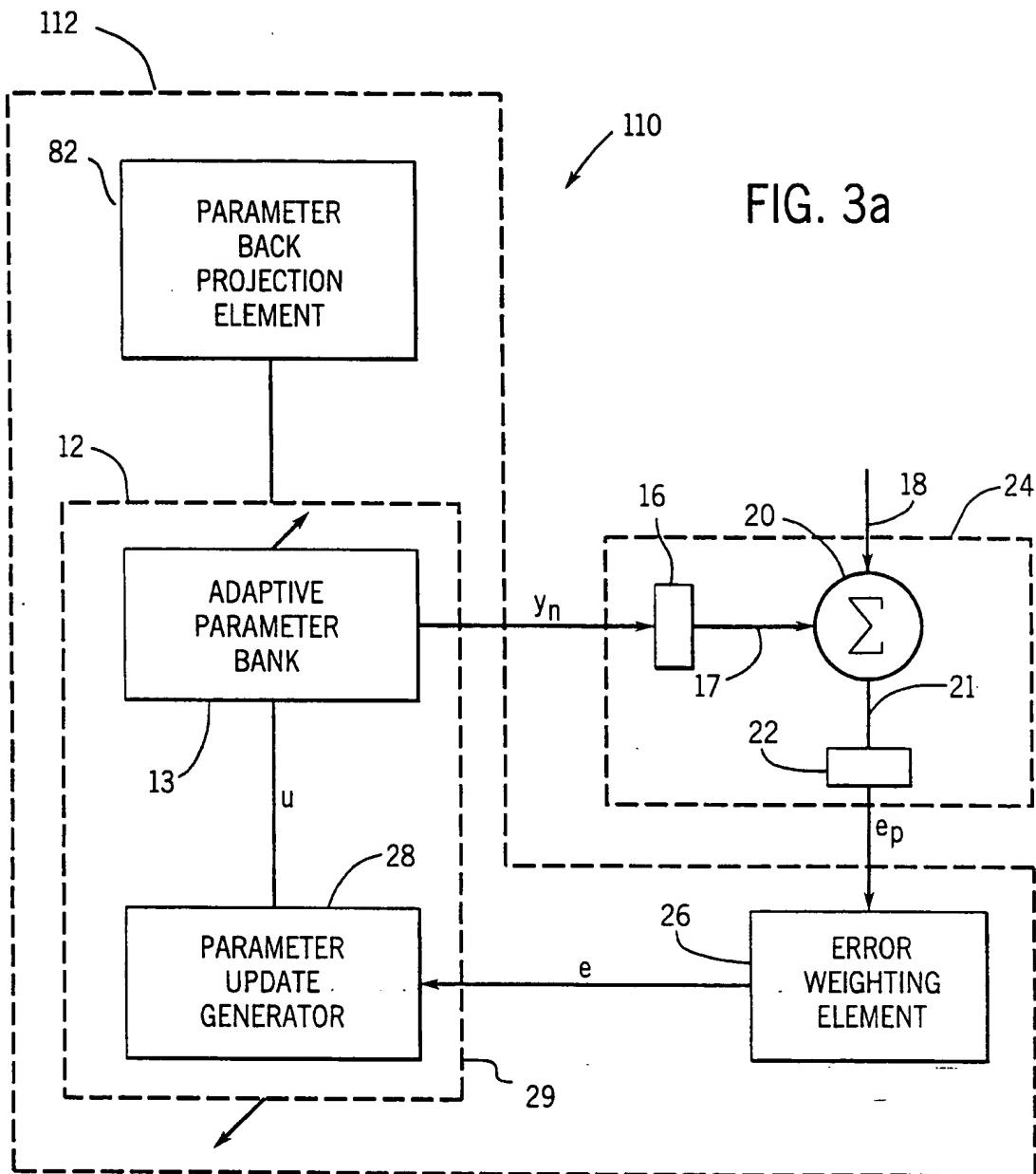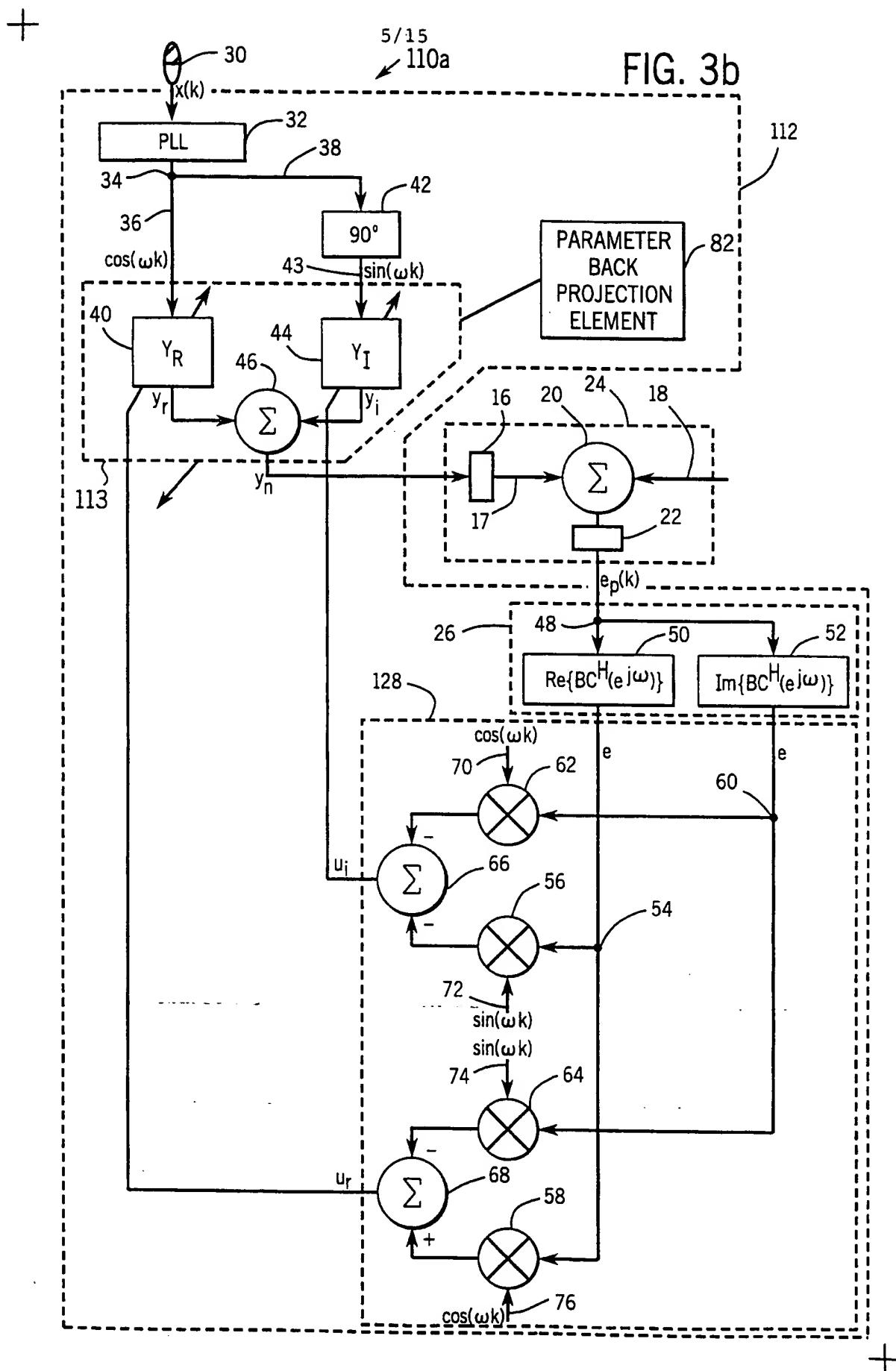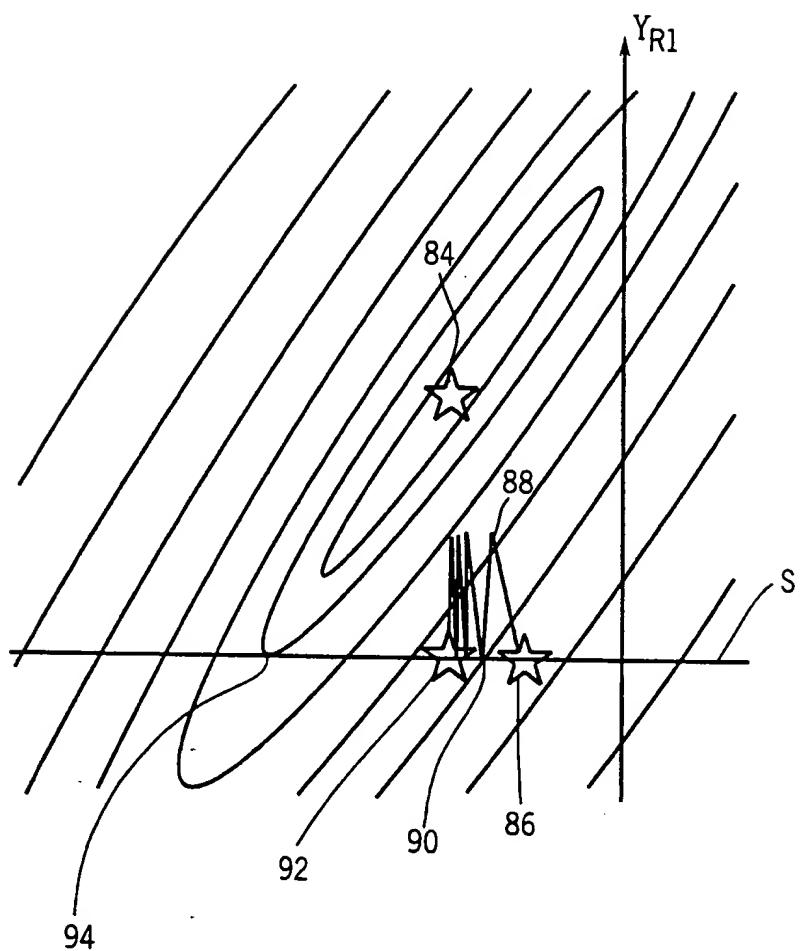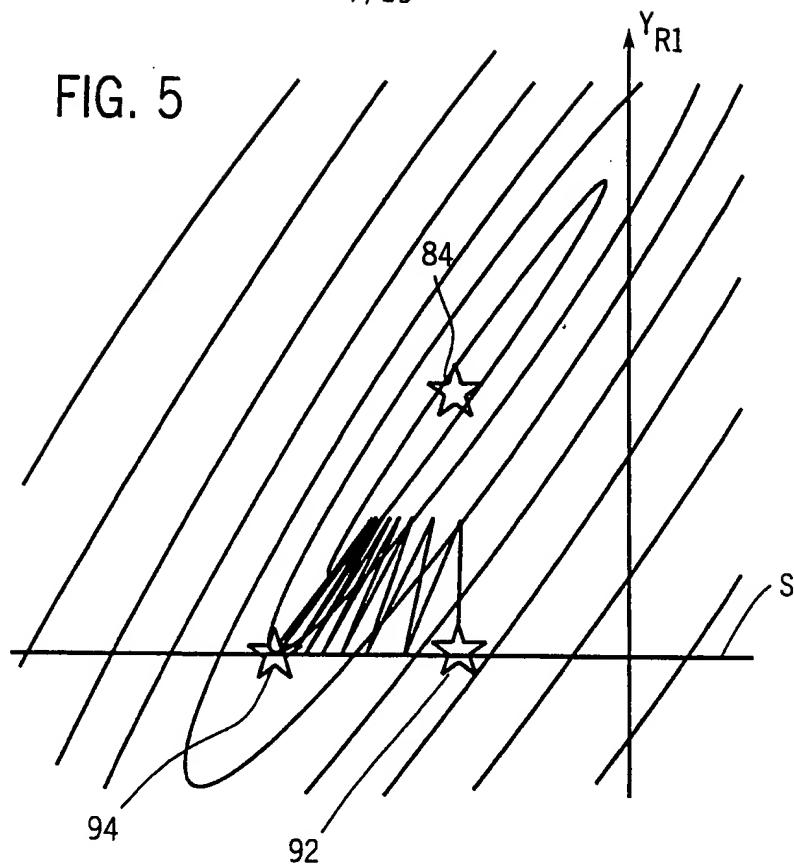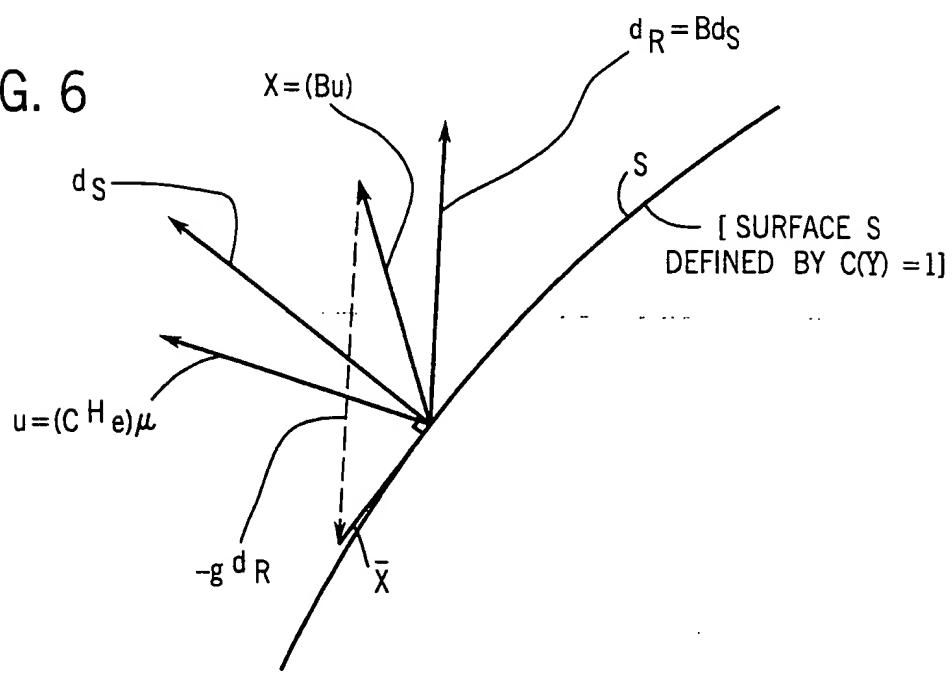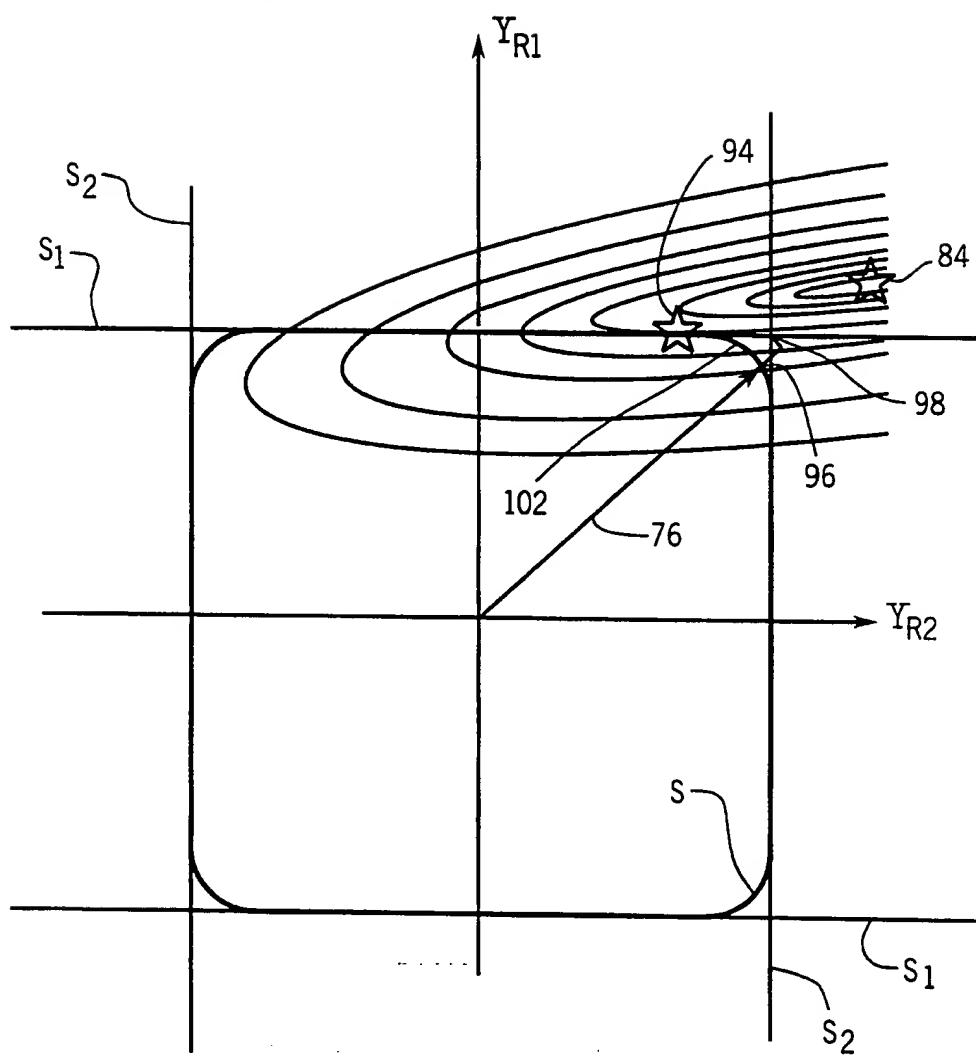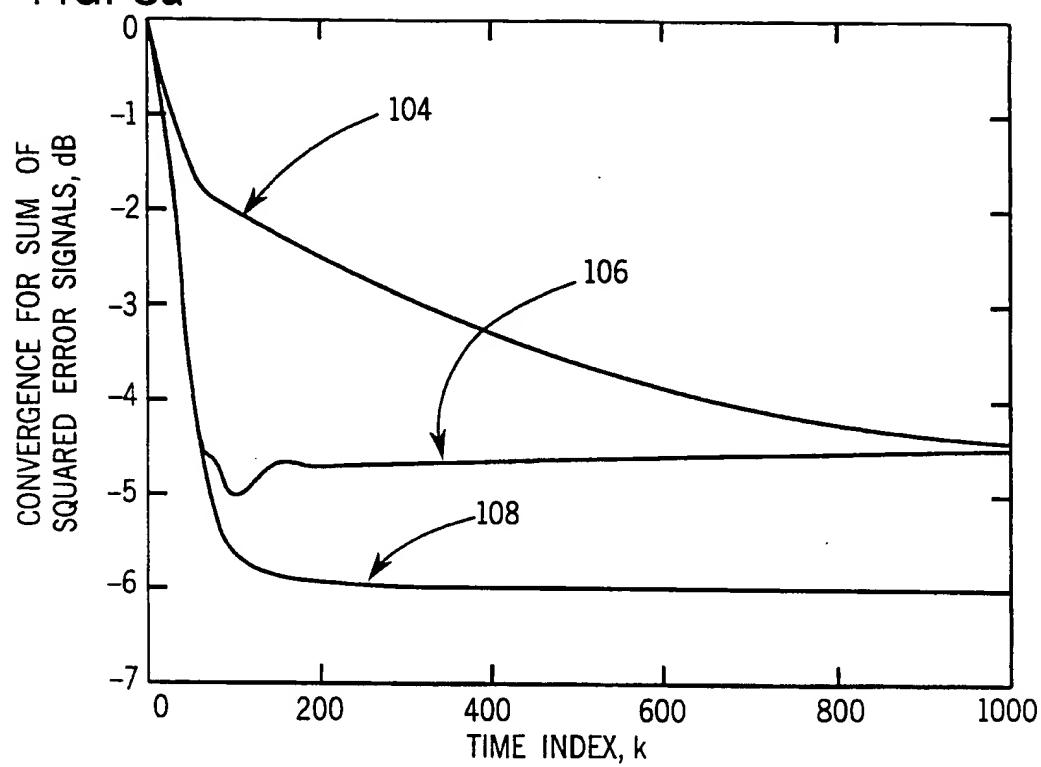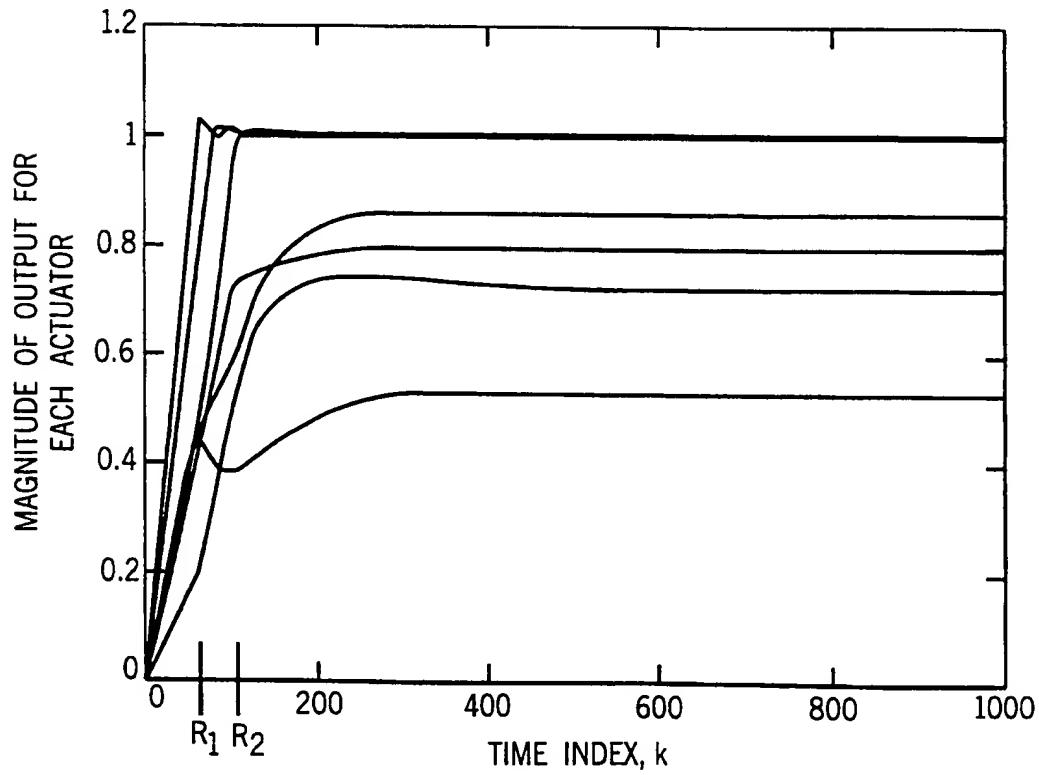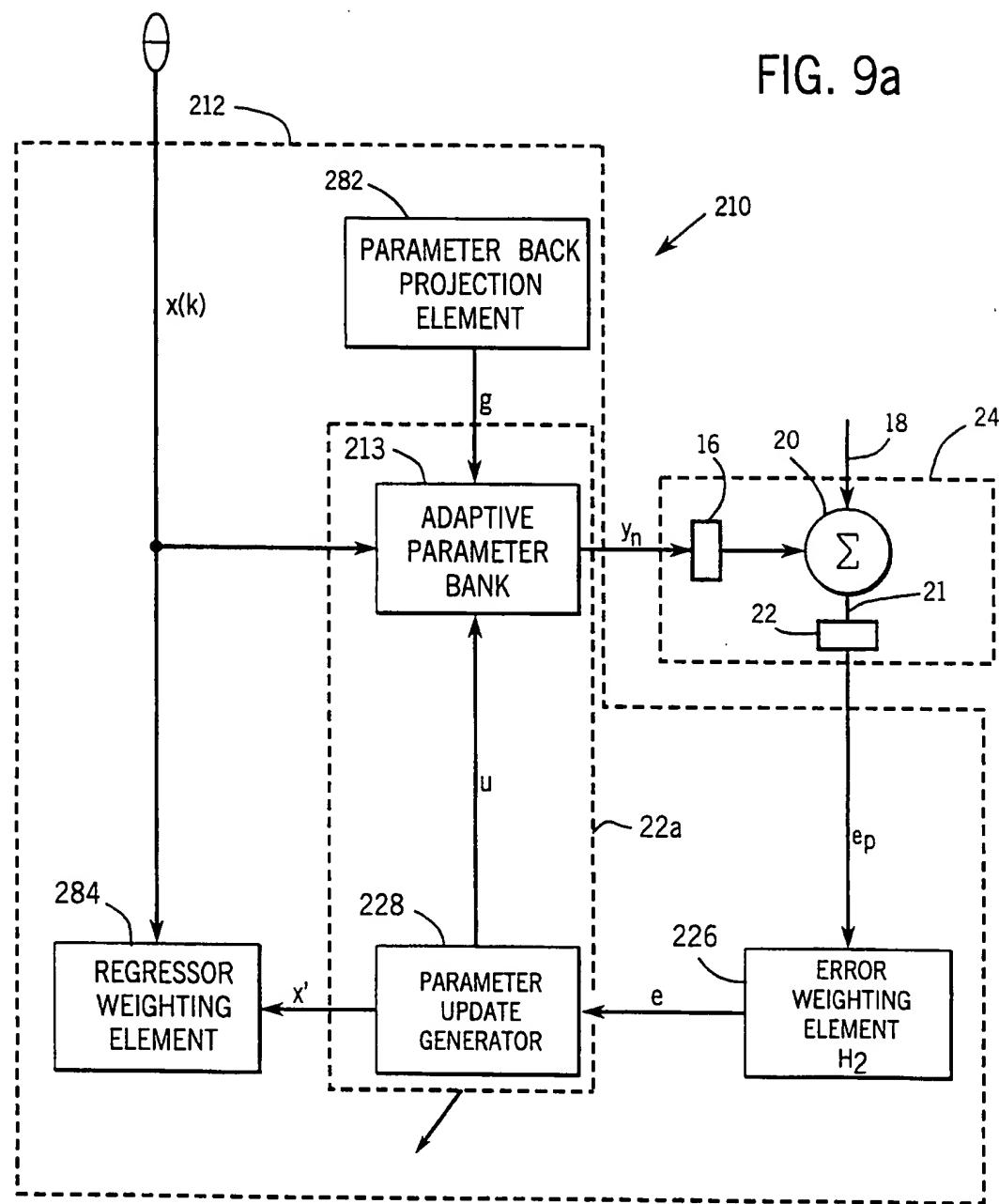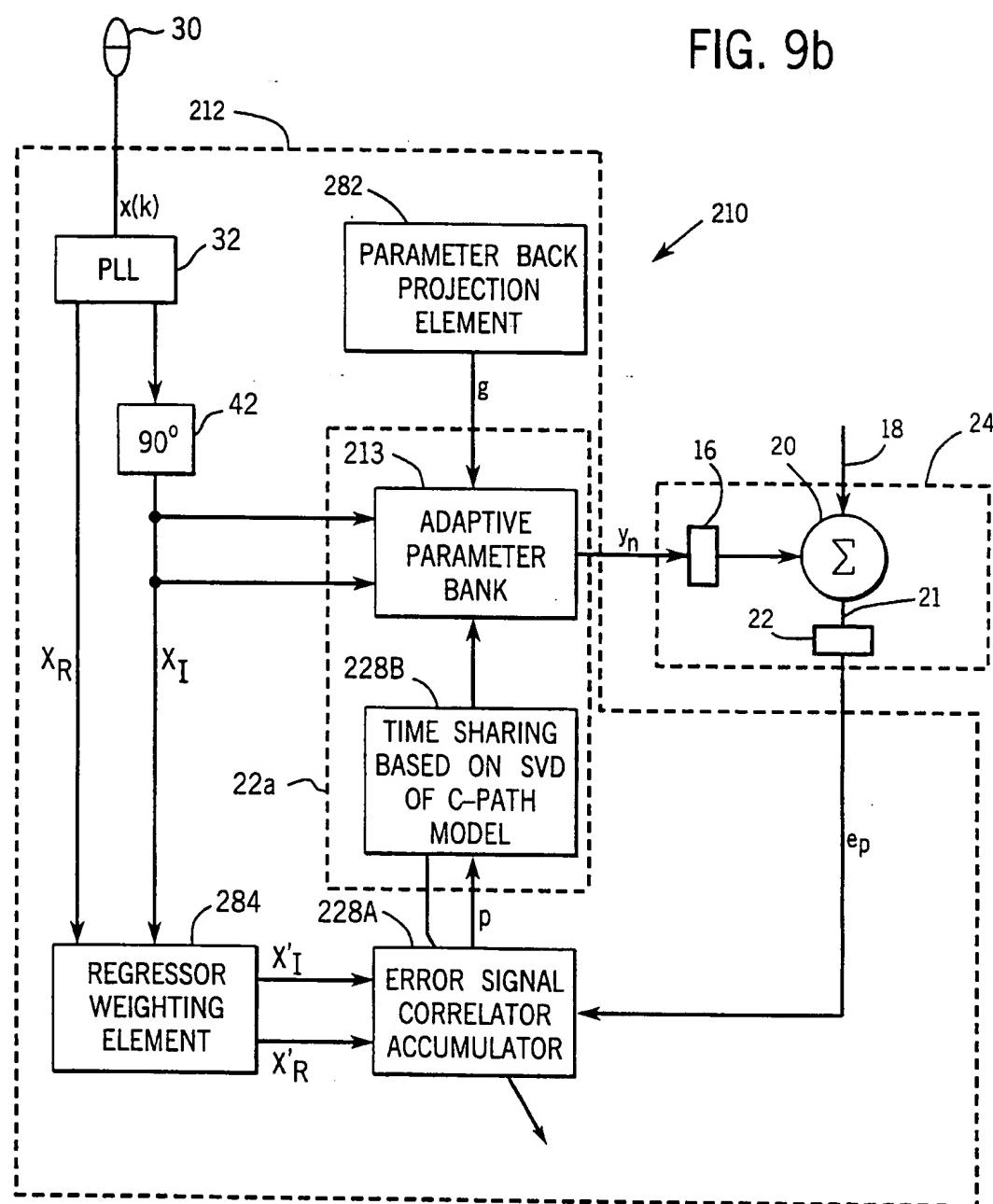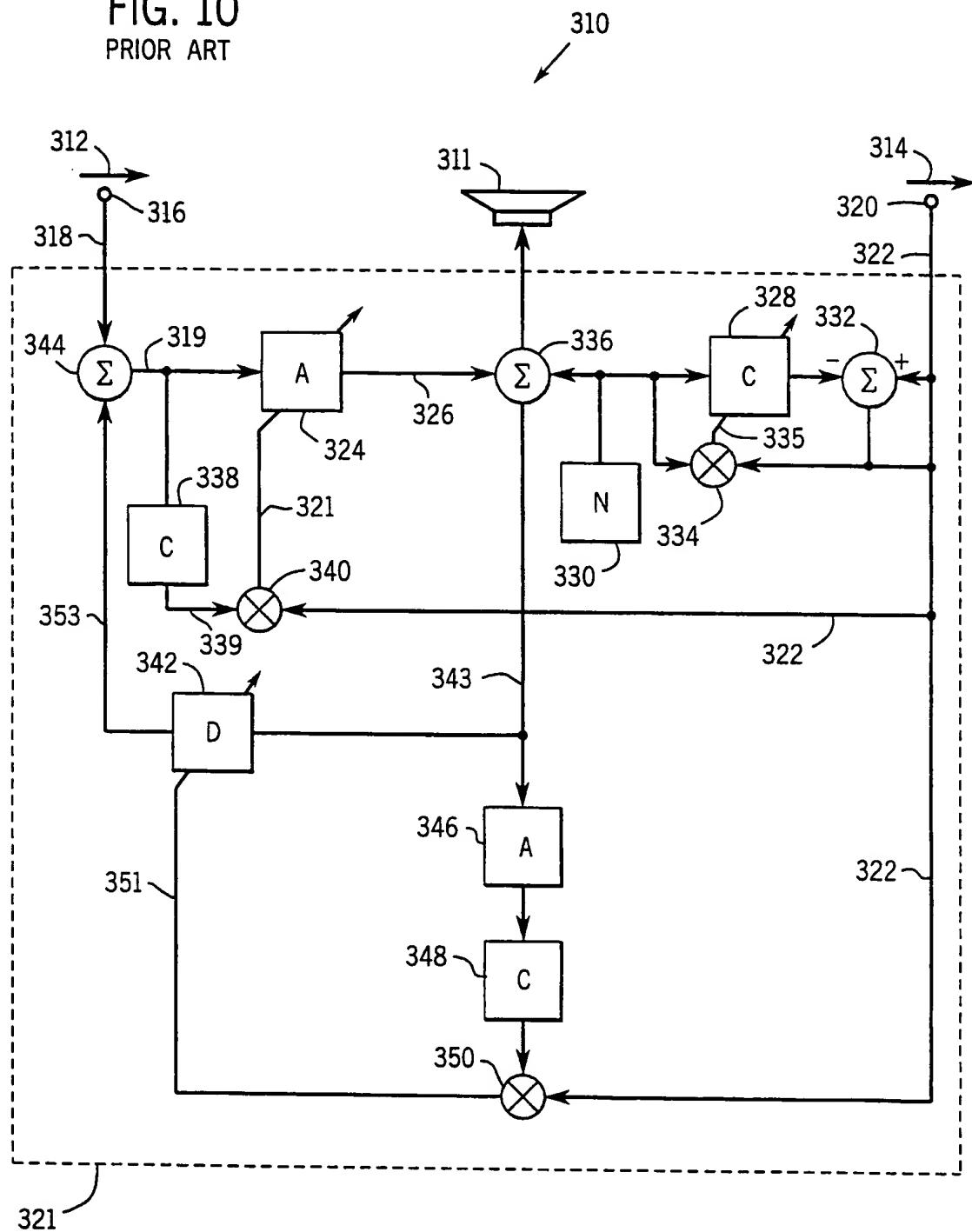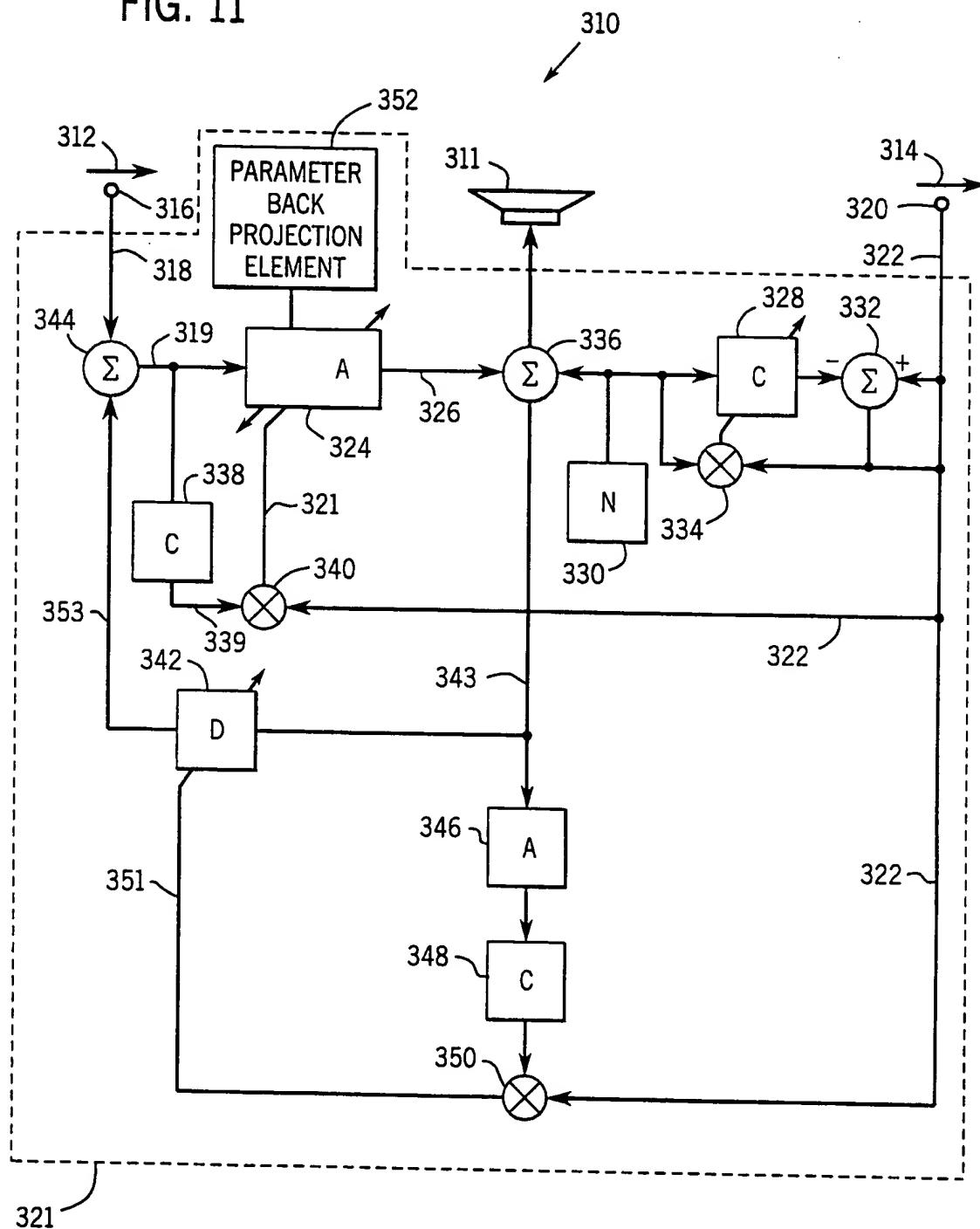
FIG. 12
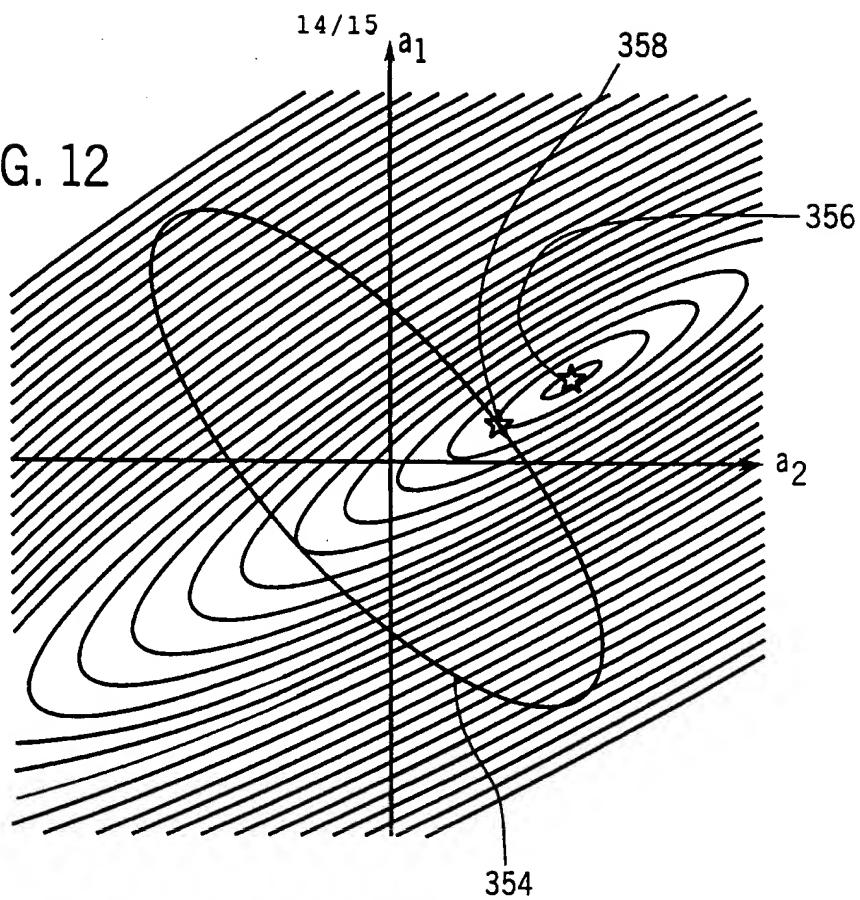
FIG. 13
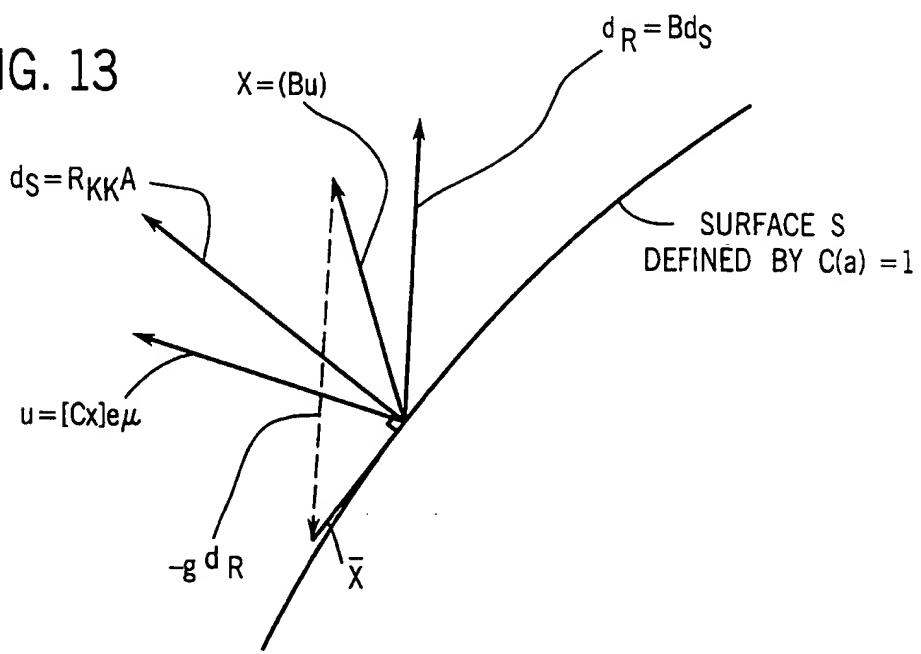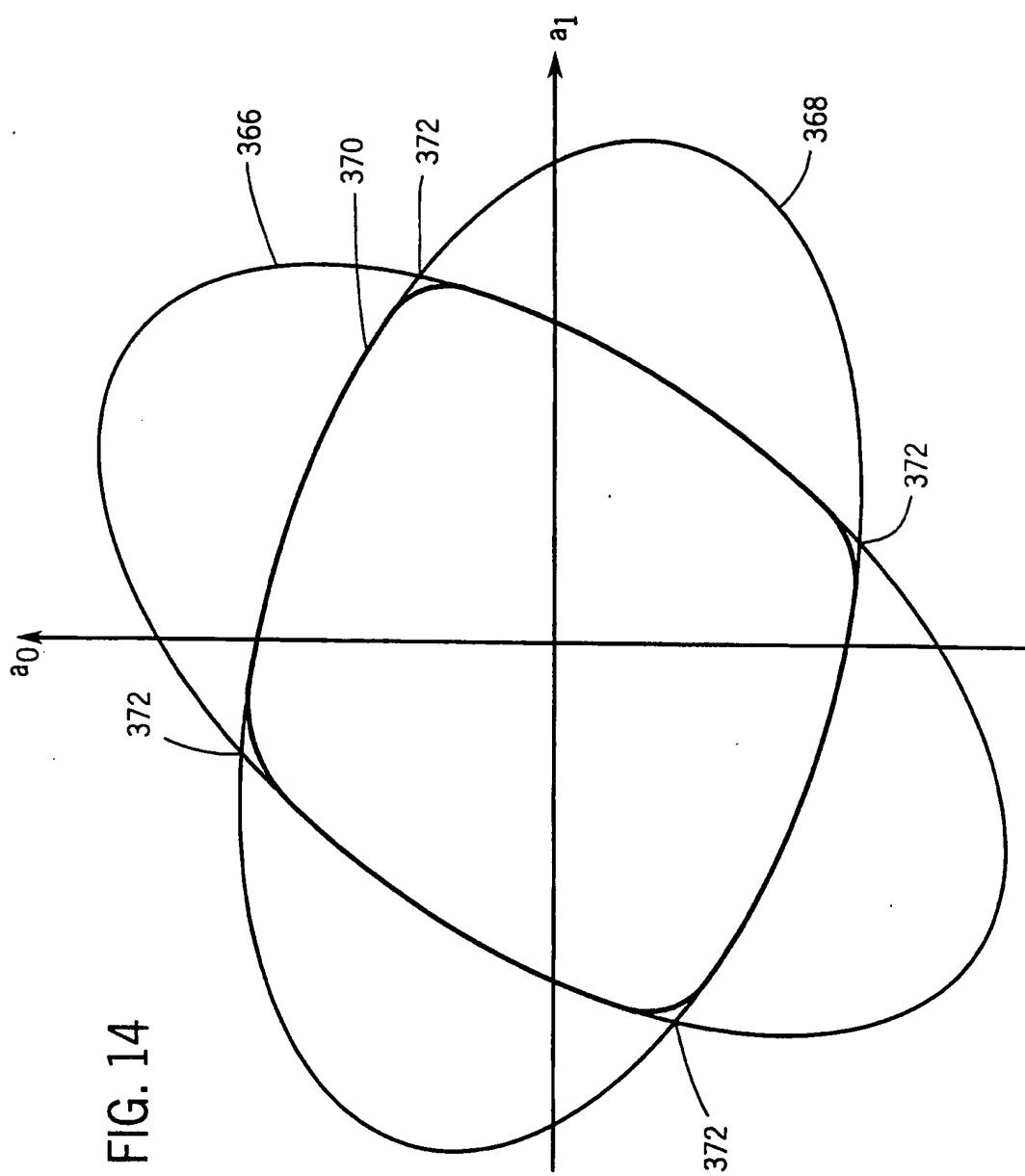
FIG. 14

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER
IPC 6   G10K11/178

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 6   G10K

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	EP 0 759 606 A (DIGISONIX INC) 26 February 1997	36-39
A	see abstract see column 1, line 45 - column 2, line 42 see column 5, line 54 - column 7, line 51 see claims 1-3,54,57 see figures 7,8 ---	1
X	EP 0 492 680 A (GEN MOTORS CORP) 1 July 1992 see abstract; figures 5,6 see claim 5 ---	36
	-/--	

[X] Further documents are listed in the continuation of box C.   [X] Patent family members are listed in annex.

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

"E" earlier document but published on or after the international filing date

"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
22 December 1998	13/01/1999

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL - 2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	de Heering, Ph.

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INTERNATIONAL SEARCH REPORT

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
X	WO 94 24662 A (STANFORD RES INST INT) 27 October 1994 see abstract see page 3, line 17 - page 4, line 32 see figures 3-5,7 ---	36-40, 52, 54-57,63
A	US 5 627 896 A (SOUTHWARD STEVE C ET AL) 6 May 1997 cited in the application see the whole document ---	1
X	US 5 633 795 A (POPOVICH STEVEN R) 27 May 1997 cited in the application	52
A	see abstract see figures 2,4 see column 1, line 47 - column 2, line 19 see claim 1 ---	5
A	WO 94 29846 A (CATERPILLAR INC) 22 December 1994 ---	
A	US 4 677 676 A (ERIKSSON LARRY J) 30 June 1987 -----	

INTERNATIONAL SEARCH REPORT

Information on patent family members

Patent document cited in search report		Publication date	Patent family member(s)		Publication date
EP 0759606	A	26-02-1997	US	5715320 A	03-02-1998
			CA	2183669 A	22-02-1997
EP 0492680	A	01-07-1992	DE	69111402 D	24-08-1995
			DE	69111402 T	04-01-1996
			US	5386472 A	31-01-1995
WO 9424662	A	27-10-1994	NONE		
US 5627896	A	06-05-1997	GB	2290635 A	03-01-1996
US 5633795	A	27-05-1997	CA	2166500 A	07-07-1996
			EP	0721179 A	10-07-1996
WO 9429846	A	22-12-1994	AU	671552 B	29-08-1996
			AU	7014594 A	03-01-1995
			EP	0655157 A	31-05-1995
			JP	8500457 T	16-01-1996
US 4677676	A	30-06-1987	AU	590384 B	02-11-1989
			AU	6860487 A	13-08-1987
			CA	1281294 A	12-03-1991
			DE	3774587 A	02-01-1992
			EP	0233717 A	26-08-1987
			JP	2539812 B	02-10-1996
			JP	62193310 A	25-08-1987